

THE ENVIRONMENTAL IMIMPACT AND SUSTAINABILITY  
APPLIED GENERAL EQUILIBRIUM (ENVISAGE) MODEL

Version 10.01

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## **Abstract**

This document's main purpose is to provide a full description of the ENVISAGE Model. ENVISAGE is a global recursive dynamic computable general equilibrium model developed originally at the World Bank. This current version reflects a complete re-coding of the model—initially done and supported by the Environment Directorate of the OECD. [Note that the model is still under development and does not yet contain all of the features of the original ENVISAGE model.] ENVISAGE has been developed to assess the interactions between economies and the global environment as affected by human-based emissions of greenhouse gases. At its core, ENVISAGE is a relatively standard recursive dynamic multi-sector multi-region CGE model. It has been complemented by an emissions and climate module [tbd] that links directly economic activities to changes in global mean temperature. And it incorporates a feedback loop that links changes in temperature to impacts on economic variables such as agricultural yields or damages created by sea level rise [tbd]. One of the overall objectives of the development of ENVISAGE has been to provide a greater focus on the economics of climate change for a more detailed set of developing countries as well as greater attention to the potential economic damages. The model remains a work in progress as there are several key features of the economics of climate change that are planned to be incorporated.

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# Chapter 1

## Introduction

The purpose of this document is to provide a complete specification of the equations of the ENVIRONMENTAL IMPACT AND SUSTAINABILITY APPLIED GENERAL EQUILIBRIUM (ENVISAGE) MODEL. The ENVISAGE Model is designed to analyze a variety of issues related to the economics of climate change:

- Baseline emissions of CO<sub>2</sub> and other greenhouse gases
- Impacts of climate change on the economy
- Adaptation by economic agents to climate change
- Greenhouse gas mitigation policies—taxes, caps and trade
- The role of land use in future emissions and mitigation
- The distributional consequences of climate change impacts, adaptation and mitigation—at both the national and household level.

ENVISAGE is intended to be flexible in terms of its dimensions. The core database—that includes energy volumes and CO<sub>2</sub> emissions—is the GTAP database, currently version 9.0 with a 2011 base year. The latter divides the world into 140 countries and regions, of which 120 are countries and the other region-based aggregations.<sup>1</sup> The database divides global production into 57 sectors—with extensive details for agriculture and food and energy (coal mining, crude oil production, natural gas production, refined oil, electricity, and distributed natural gas). Appendix ?? provides more detail. Due to numerical and algorithmic constraints, a typical model is limited to some 20-30 sectors and 20-30 regions.

This document describes the current version of ENVISAGE, which is still in a developmental stage. This current version includes the following:

- Capital vintage production technology that permits analysis of the flexibility of economies
- Partially endogenous technical change [tbd]
- A detailed specification of energy demand in each economy
- Incorporation of a limited set of new energy technologies

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<sup>1</sup> The countries defined in GTAP cover well over 90 percent of global GDP and population.



- A resource depletion module for coal, oil and natural gas [tbd]
- The ability to introduce future alternative energy (or backstop) technologies [tbd]
- CO<sub>2</sub> emissions that are fuel and demand specific
- Incorporation of the main Kyoto greenhouse gases (methane, nitrous oxide and the fluoridated gases)
- A flexible system for incorporating any combination of carbon taxes, emission caps and tradable permits
- A simplified climate module that links greenhouse gas emissions to atmospheric concentrations combined with a carbon cycle that leads to radiative forcing and temperature changes. [tbc]
- A water module that incorporates water use for irrigation by crop, and aggregate water demand in other parts of the economy.

The current work program includes the following tasks:

- Addition of marginal abatement cost curves for the non-CO<sub>2</sub> gases
- Adding a more detailed land-use module

The ENVISAGE Model is a descendant of a family of models that originated at the OECD in the late 1980s and early 1990s, WALRAS, RUNS and GREEN.<sup>2</sup> ENVISAGE was initially developed at the World Bank in 2007 and was a re-coded version of the World Bank’s LINKAGE model ([van der Mensbrugge \(2011\)](#)), which had a trade focus. It is designed specifically to analyze climate change issues and thus incorporates a more developed energy sector, a climate module (that makes integrated assessment an option), and climate change impact feedbacks. ENVISAGE is coded using the GAMS/MCP package.

The current (draft) version is a complete re-coding of the original ENVISAGE Model largely undertaken at the behest of the OECD’s Environment Directorate, whose ENV-Linkages model also descended from the above-mentioned OECD models. [The current version has yet to incorporate all of the features of the original ENVISAGE Model, but work is ongoing to complete the task.] This version, developed and residing at the Center for Global Trade Analysis (GTAP), thus shares a core code with the OECD ENV-Linkages model, though further development of the individual models will diverge some over time due to differing priorities and preferences. The version developed for the OECD, ENV-Linkages, is a relatively faithful translation of their original specification ([Chateau, Dellink, and Lanzi \(2014\)](#)), but with a clean interface, streamlined code and improvements in its numerical properties. The water module was developed specifically for the OECD. Note that the ENV-Linkages team at the OECD was a core contributor to the quantification of the Shared Socio-Economic Pathways that will frame the analysis of the economics of climate change for at least the next decade ([Dellink et al. \(2017\)](#)).

The next section provides a brief summary of the main model features. This is followed by a much longer section with a full-blown description of the ENVISAGE Model.

<sup>2</sup> OECD (1989/1990), [Burniaux \(1987\)](#), [Burniaux and van der Mensbrugge \(1994\)](#), [Burniaux, Nicoletti, and Oliveira-Martins \(1992\)](#), and [van der Mensbrugge \(1994\)](#). Note that WALRAS and RUNS were initially developed at Stanford University and the Université Libre de Bruxelles, respectively.

## Chapter 2

# The Envisage Model in a nutshell

The ENVISAGE Model at its core is a recursive dynamic and global computable general equilibrium model (CGE). It follows the circular flow of an economy paradigm. Firms purchase input factors (for example labor and capital) to produce goods and services. Households receive the factor income and in turn demand the goods and services produced by firms. And equality of supply and demand determine equilibrium prices for factors, goods and services. The model is solved as a sequence of comparative static equilibria where the factors of production are exogenous for each time period and linked between time periods with accumulation expressions.

Production is implemented as a series of nested constant-elasticity-of-substitution (CES) functions the aim of which is to capture the substitutability of all inputs. Three production archetypes are implemented. The first is for crops that reflects intensification of inputs versus land extensification. The second is for livestock that reflects range-fed versus ranch-fed production. The final, also referred to as the default, revolves largely around capital/labor substitutability. Some production activities highlight specific inputs (for example agricultural chemicals in crops and feed in livestock) and all activities include energy and its components as part of the cost minimization paradigm. Production is also identified by vintage—divided into *Old* and *New*—with typically lower substitution possibilities associated with *Old* capital.

Each production activity is allowed to produce more than one commodity—for example the ethanol sector can produce ethanol and distiller’s dried grains with solubles (DDGS). And commodities can be formed by the output of one or more activities (for example electricity). ENVISAGE therefore uses a different classification of activities and commodities.<sup>1</sup> One of the features of the model is that it integrates the new GTAP power data base that disaggregates GTAP’s electricity sector (‘ely’) into 11 different power sources plus electricity transmission and distribution. Though the database has both the supply and demand side for all 11 power sources, the aggregation facility permits the aggregation of electricity demand into a single commodity and the ‘make’ matrix specification combines the output from the different power activities into a single electricity commodity.

Income accrues from payments to factors of production and is allocated to households (after taxes). The government sector accrues all net tax payments and purchases goods and services. The model incorporates multiple utility functions for determining household demand. There is a set of three household demand functions linked to the ubiquitous linear expenditure system (LES): the standard LES, the extended LES (ELES) that incorporates household saving into the utility function, and ‘an implicitly directly additive demand system’ (AIDADS), that allows for non-linear Engel curves in the LES framework. The fourth option uses the constant differences in elasticity (CDE) utility function that is used in the core GTAP model (Hertel (1997)). The ELES

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<sup>1</sup> Production activities are indexed with  $a$  and commodities are indexed with  $i$ .

incorporates the decision to save in a top level utility function. The other demand systems assume savings is an exogenous proportion of disposable income. The consumer utility function determines consumer demand bundles that are subsequently converted to produced goods using a consumer demand 'make' or transition matrix. Investment is savings driven and equal to domestic saving adjusted by net capital flows.

Trade is modeled using the so-called Armington specification that posits that demand for goods are differentiated by region of origin. The model allows for domestic/import sourcing at the aggregate level (after aggregating domestic absorption across all agents), or at the agent-level. A second Armington nest allocates aggregate import demand across all exporting regions. Irrespective of the specification of the top level sourcing between domestic goods and the aggregate import bundle, the second level Armington nest is done at the aggregate level, i.e. it is not agent specific. Exports are modeled in an analogous fashion using a nested constant-elasticity-of-transformation (CET) specification. The domestic supply of each commodity is supplied to the domestic market and an aggregate export bundle using a top-level CET function. The latter is allocated across regions of destination using a second-level CET function.<sup>2</sup> Each bilateral trade node is associated with four prices: 1) the producer price; 2) the export border price, also referred to as the free-on-board (FOB) price; 3) the import border price, also referred to as the cost, insurance and freight (CIF) price; and 4) the end-user price that includes all applicable trade taxes. The wedge between the producer price and the FOB price is represented by the export tax (or subsidy if negative) and the wedge between the CIF and end-user prices represents the import tariff (and perhaps other import related distortions). The wedge between the CIF and FOB prices represents the international trade and transport margins. These margins represent the use of real resources that are supplied by each region. The global international trade and transport sector purchases these services from each region so as to minimize the aggregate cost.

The model has two fundamental markets for goods and services. Domestically produced goods sold on the domestic market, and domestically produced goods sold by region of destination. All other goods and services are composite bundles of these goods. Two market equilibrium conditions are needed to clear these two markets.<sup>3</sup>

The model incorporates five types of production factors: 1) labor (of which there can be up to 5); 2) capital; 3) land; 4) a sector specific natural resource (such as fossil fuel energy reserves); and 5) water. The labor market is allowed to be segmented (though not required). The model allows for regime switching between full and partial wage flexibility. Capital is allocated across sectors so as to equalize rates of returns. If all sectors are expanding, *Old* capital is assumed to receive the economy-wide rate of return. In contracting sectors, *Old* capital is sold on secondary markets using an upward sloping supply curve. This implies that capital is only partially mobile across sectors. Aggregate land and water supply are specified using supply curves. Though there are several options, the preferred supply curve is a logistic function that has an upper bound. Water demand also includes exogenous components for environmental uses and groundwater recharge. Land and water are allocated across activities using a nested CET specification.<sup>4</sup> Natural resources are supplied to each sector using an iso-elastic supply function.

The model incorporates the main greenhouse gases—carbon, methane, nitrous oxides and fluorinated gases. Emissions are generated by consumption of commodities (such as fuels), factor use

<sup>2</sup> The model allows for perfect transformation, which is the standard specification in the GTAP model.

<sup>3</sup> If there are  $N$  commodities and  $R$  regions, there will be  $R \times N$  market clearing conditions for domestic goods and  $R \times N \times R$  market clearing conditions for bilateral trade.

<sup>4</sup> Land is only implemented for agricultural activities. Water demand by activity is only present in irrigated crop sectors. Other water demand is based on aggregate demand functions with market clearing, but is not part of the cost structure.

(for example land in rice production and herds in livestock production) and there are also processed base emissions such as methane from landfills.<sup>5</sup> A number of carbon control regimes are available in the model. Carbon taxes can be imposed exogenously—potentially differentiated across regions. The incidence of the carbon tax allows for partial or full exemption by commodity and end-user. For example households can be exempted from the carbon tax on natural gas consumption. The model allows for emission caps in a flexible manner—where regions can be segmented into coalitions on a multi-regional or global basis. In addition to the standard cap system, a cap and trade system can be defined where each region within a coalition is assigned an initial emission quota.

Dynamics involves three elements. Labor supply (by skill level) grows at an exogenously determined rate. The aggregate capital supply evolves according to the standard stock/flow motion equation, i.e. the capital stock at the beginning of each period is equal to the previous period's capital stock, less depreciation, plus the previous period's level of investment. The third element is technological change. The standard version of the model assumes labor augmenting technical change—calibrated to given assumptions about GDP growth and inter-sectoral productivity differences. In policy simulations, technology is typically assumed to be fixed at the calibrated levels.

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<sup>5</sup> The current version of the model does not include carbon emissions from deforestation—an important source of global carbon emissions.

## Chapter 3

# Model specification

The next section describes the key model dimensions. Subsequent sections describe the various blocks or modules of the model using the traditional circular flow scheme of economics, i.e. starting with production and factor incomes, income distribution, demand, trade, and macro closure. At the end, there is a discussion on the model dynamics.

### 3.1 Model dimensions

The model specification in GAMS is mostly as general as possible, i.e. virtually all variables and equations are specified using set-based indices that are defined at run-time when the data is read in. The model can thus be configured for any number of regions (up to 140 with the latest GTAP release) and any number of sectors (up to 57 with the standard GTAP database and 76 with the power/water database).

Table 3.1 lists the main indices used by the model. On the demand side the key dimension is the number of Armington agents (indexed by  $aa$ ). The Armington agents include all production activities (indexed by  $a$ ), and final demand agents (indexed by  $fd$ ). The latter are split into households ( $h$ ), government ( $gov$ ) and investment ( $gov$ ). Activities, i.e. production sectors, are separated from commodities. The former are indexed by  $a$  and the latter by  $i$  (with a possible alias of  $j$ ). Thus intermediate demand,  $XA_{i,a}$  represents the demand for commodity  $i$  by activity  $a$ . In the standard GTAP database there is a one-to-one mapping between activities and commodities. The ENVISAGE model allows for a non-diagonal make matrix that is user-determined at the time of aggregating the GTAP database. This has proven useful, for example, for the power sector. The new power database has 11 power generating technologies with a complementary transmission and distribution activity. These can be aggregated to a single electricity commodity using the make matrix (with a nested CES specification).

The model incorporates three different production structures—crops, livestock and other. These will be indexed by  $acr$ ,  $alv$  and  $ax$ , respectively—all subsets of  $a$ . Activities are also sub-divided into two 'geographic' zones—rural and urban. Typically, agricultural activities are assigned to the rural zone and all other activities to the urban zone. The model allows for segmented labor markets by geographic zone with a Harris-Todaro-type migration function (Harris and Todaro (1970)). Most of the other indices are relatively self-explanatory and will be further elaborated on in the model description.

Three key indices are not part of the table—an index for power ( $pb$ ), land ( $lb$ ) and water bundles ( $wbnd$ ). These indices are user-defined and are used to aggregate power supply in the power module, allocate land across sectors in the land supply module and allocate water across

uses in the water supply module. They will be further described in their respective sections.

Table 3.1: Sets used in model definition

Set	Description
<i>aa</i>	Armington agents
<i>a</i>	Activities (a subset of <i>aa</i> )
<i>acr</i>	Crop activities (a subset of <i>a</i> )
<i>alv</i>	Livestock activities (a subset of <i>a</i> )
<i>ax</i>	All other activities (a subset of <i>a</i> )
<i>elya</i>	Power activities (a subset of <i>a</i> )
<i>z</i>	Set of zones—just rural ( <b>rur</b> ) and urban ( <b>urb</b> )
<i>i</i>	Produced (or supplied) goods
<i>inum</i>	Set of manufacturing sectors (subset of <i>i</i> , used in definition of numéraire)
<i>fp</i>	Factors of production
<i>l</i>	Labor categories (subset of <i>fp</i> )
<i>ul</i>	Unskilled labor categories (subset of <i>l</i> )
<i>sl</i>	Skilled labor categories (subset of <i>l</i> )
<i>cap</i>	Capital account (subset of <i>fp</i> )
<i>lnd</i>	Land account (subset of <i>fp</i> )
<i>nrs</i>	Natural resource account (subset of <i>fp</i> )
<i>wat</i>	Water account (subset of <i>fp</i> )
<i>k</i>	Consumed commodities
<i>nrg(k)</i>	The energy bundle in consumed commodities
<i>fd</i>	Final demand (subset of <i>aa</i> )
<i>fdc</i>	Final demand excluding households (subset of <i>fd</i> )
<i>h</i>	Households (subset of <i>fd</i> )
<i>gov</i>	Government account (subset of <i>fd</i> )
<i>inv</i>	Investment account (subset of <i>fd</i> )
<i>gy</i>	Government revenue accounts
<i>itax</i>	Indirect taxes (subset of <i>gy</i> )
<i>ptax</i>	Production tax account (subset of <i>gy</i> )
<i>mtax</i>	Import tariff account (subset of <i>gy</i> )
<i>etax</i>	Export tax account (subset of <i>gy</i> )
<i>vtax</i>	Tax on factors of production account (subset of <i>gy</i> )
<i>ptax</i>	Production tax account (subset of <i>gy</i> )
<i>ctax</i>	Carbon tax (subset of <i>gy</i> )
<i>dtax</i>	Direct taxes (subset of <i>gy</i> )
<i>r</i>	Regions
<i>s, d</i>	Aliases with <i>r</i> (for <i>source</i> and <i>destination</i> regions)
<i>rnum</i>	Set of regions used in definition of numéraire (subset of <i>r</i> )
<i>rres</i>	Residual region (subset of <i>r</i> , must be of single dimension)
<i>em</i>	Emission types

### 3.2 Production block

Production is implemented using a nested CES structure—rather standard in many applied general equilibrium models. Three production structure prototypes are introduced—crops, livestock and all other (the default production structure). Figures 1, 2 and 3 depict the production structures for each of the three prototypes. These figures are supplemented by the nesting for the so-called *KEF* and energy bundles in respectively figures 4 and 5. This section will provide a full description of all the nests starting from the top nest. At the ends of each of the terminal nodes will be the derived demand for the basic components (i.e. inputs) of production—intermediate goods and factors of production.

Each nest is re-produced for each of the possible vintages. In the comparative static version of the model there is usually only a single vintage. The default in the dynamic version of the model is to have two vintages—so-called *Old* vintage (or installed capital), and *New* that represents the most recent supply of capital. The former is meant to be only partially mobile across sectors, whereas the latter is fully mobile—detailed further below.

Production for each vintage is thus associated with a unit (or marginal) cost of production, represented by  $uc$ . The post tax unit cost of production,  $PXv$ , is equal to the tax-adjusted pre-tax unit cost of production, equation (P-1), where  $\tau^{uc}$  is the tax on the cost of production. The aggregate unit cost of production across vintages,  $PX$ , is given in equation (P-2) and is the weighted sum of the vintage-specific costs of production, with weights given by the production volume shares where  $XPv$  and  $XP$  represent respectively output by vintage and aggregate output. The output (or market) price,  $PXv$ , is equal to the unit cost of production, to which is added a markup,  $\pi^m$ ,<sup>1</sup> and adjusted by an output tax represented by  $\tau^p$ , equation (P-3).

$$PXv_{r,a,v} = UC_{r,a,v} (1 + \tau_{r,a,v}^{uc}) \quad (\text{P-1})$$

$$PX_{r,a} XP_{r,a} = \sum_v PXv_{r,a,v} XPv_{r,a,v} \quad (\text{P-2})$$

$$PP_{r,a} = (PX_{r,a} + \pi_{r,a}^m) (1 + \tau_{r,a}^p) \quad (\text{P-3})$$

The remainder of this section describes the various CES nests that represent the production structure by vintage. The purpose of these CES nests is to replicate the substitution and complementarity relations across all of the inputs. The top level CES nest represents the combination of output,  $XPX$ , with a bundle of non-CO<sub>2</sub> greenhouse gas (GHG) emissions,  $XGHG$ . The latter is a special bundle that is used to simulate the marginal cost of abating non-CO<sub>2</sub> greenhouse gases. As the price of this bundle increases (e.g. from a tax on emissions), producers substitute away from this relatively more costly input. Equations (P-4) and (P-5) represent the derived demands for the output and GHG bundles respectively. These are the standard CES demand functions where  $PXP$  and  $PXGHG$  represent the price of the component bundles and  $UC$  is the price of the composite (or aggregate bundle). In this case,  $UC$ , is the unit cost of production, including the price associated with the GHG emissions. The parameters  $\alpha^{xp}$  and  $\alpha^{ghg}$  represent the standard CES share parameters,  $A^{xp}$  is a tech-neutral shift in the production nest and  $\sigma^{xp}$  is the elasticity of substitution between production and the emissions of GHGs. The production nest also allows for input specific technological change as represented by the parameters  $\lambda^{xp}$  and  $\lambda^{ghg}$ . These are

<sup>1</sup> For the moment always exogenous and typically set to 0.

typically exogenous. Equation (P-6) defines the component price of the CES bundle,  $UC$ . The expression uses the CES dual price formula and could be replaced by the zero-profit condition.<sup>2</sup>

$$XPX_{r,a,v} = \alpha_{r,a,v}^{xp} \left( A_{r,a,v}^{xpv} \lambda_{r,a,v}^{xp} \right)^{\sigma_{r,a,v}^{xp}-1} \left( \frac{UC_{r,a,v}}{PXP_{r,a,v}} \right)^{\sigma_{r,a,v}^{xp}} XPv_{r,a,v} \quad (\text{P-4})$$

$$XGHG_{r,a,v} = \alpha_{r,a,v}^{ghg} \left( A_{r,a,v}^{xpv} \lambda_{r,a,v}^{ghg} \right)^{\sigma_{r,a,v}^{xp}-1} \left( \frac{UC_{r,a,v}}{PXGHG_{r,a,v}} \right)^{\sigma_{r,a,v}^{xp}} XPv_{r,a,v} \quad (\text{P-5})$$

$$UC_{r,a,v} = \frac{1}{A_{r,a,v}^{xpv}} \left[ \alpha_{r,a,v}^{xp} \left( \frac{PXP_{r,a,v}}{\lambda_{r,a,v}^{xp}} \right)^{1-\sigma_{r,a,v}^{xp}} + \alpha_{r,a,v}^{ghg} \left( \frac{PXGHG_{r,a,v}}{\lambda_{r,a,v}^{ghg}} \right)^{1-\sigma_{r,a,v}^{xp}} \right]^{\frac{1}{1-\sigma_{r,a,v}^{xp}}} \quad (\text{P-6})$$

The second level nest decomposes aggregate production net of the GHG bundle into two bundles,  $ND^I$  and  $VA$ . The first bundle includes all intermediate goods with the exception of energy goods and other intermediate goods that are treated specially in a given activity. In the default configuration of the model, activity-specific intermediate goods include fertilizers for crop activities and feed for livestock activities.<sup>3</sup> The  $VA$  contains all factors of production, the energy goods and activity-specific goods where applicable. Equation (P-7) determines the demand for the top level intermediate demand bundle,  $ND^I$ . Equation (P-8) determines the demand for the  $VA$  bundle. The bundle prices are respectively  $PND^I$  and  $PVA$  and the substitution elasticity is given by  $\sigma^p$ . It is worth noting that the equation for  $ND^I$  is summed over all vintages. This is because the further decomposition of the  $ND^I$  bundle is assumed to be independent of the vintage, whereas the decomposition of the  $VA$  is vintage specific as the substitution elasticities further down the nest are allowed to vary by vintage. Equation (P-9) determines the price of  $XPX$ ,  $PXP$ .

$$ND_{r,a}^I = \sum_v \alpha_{r,a,v}^{nd1} \left( \frac{PXP_{r,a,v}}{PND_{r,a}^I} \right)^{\sigma_{r,a,v}^p} XPX_{r,a,v} \quad (\text{P-7})$$

$$VA_{r,a,v} = \alpha_{r,a,v}^{va} \left( \frac{PXP_{r,a,v}}{PVA_{r,a,v}} \right)^{\sigma_{r,a,v}^p} XPX_{r,a,v} \quad (\text{P-8})$$

$$PXP_{r,a,v} = \left[ \alpha_{r,a,v}^{nd1} (PND_{r,a}^I)^{1-\sigma_{r,a,v}^p} + \alpha_{r,a,v}^{va} (PVA_{r,a,v})^{1-\sigma_{r,a,v}^p} \right]^{\frac{1}{1-\sigma_{r,a,v}^p}} \quad (\text{P-9})$$

The middle CES nests are activity specific and reflect the three production structure prototypes described above—crops, livestock and all other activities. The crop production structure is designed to capture production characterized by the choice between intensification versus extensification, i.e. where land is abundant and cheap, production will tend to expand using more land, and vice versa if land is scarce and expensive. Livestock production is characterized by feed versus land substitution. In the default production structure, the main characteristic of production is the standard capital and labor substitution.

<sup>2</sup> The zero profit condition (without indices) is  $UC.XPv = PXP.XPX + PXGHG.XGHG$ .

<sup>3</sup> The standard GTAP database does not include fertilizers. These are associated with the 'crp' good, i.e. chemicals, rubber, and plastics. The standard feed goods are wheat, coarse grains and oil seeds, respectively 'wh't', 'gro' and 'osd' in the GTAP database.



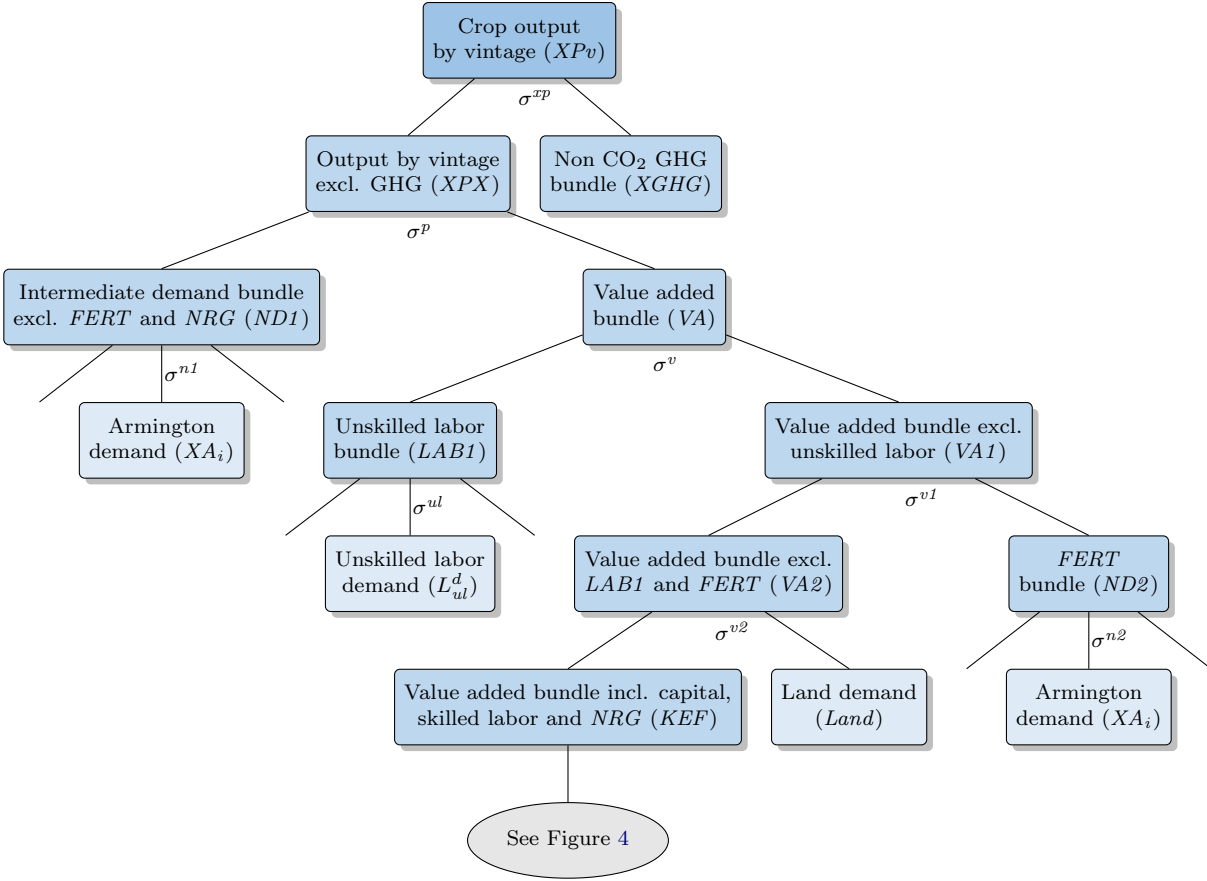


Figure 1: **Crop production nest**

These different structures will be captured by a set of intermediate bundles, represented by  $VA$  and  $VA^2$  that represent different composition of factors and activity-specific intermediate goods where appropriate. The top of the nest is represented by the bundle  $VA^1$ . The bottom of this set of nests are the bundles  $LAB^1$ ,  $Land^d$ ,  $KEF$  and  $ND^2$ , where appropriate. The bundle  $LAB^1$  represents the demand for the unskilled labor bundle.<sup>4</sup> The variable  $Land^d$  represents the activity's demand for the land factor. The bundle  $KEF$  represents the nested combination of capital, skilled labor, energy and the natural resource factor, where appropriate. The bundle  $ND^2$  represents the activity-specific demand for intermediates—fertilizers in the case of crops and feed in the case of livestock activities. The decomposition of these three bundles<sup>5</sup> is the same for all activities and described below. Table 3.2 summarily describes the composition of these middle nests for the three production prototypes. Figures 1, 2 and 3 provide an illustrative description of the production nests.

<sup>4</sup> The user determines what labor types are considered 'unskilled' labor. The user can decide to have all labor types in this bundle, in which case the skilled labor bundle,  $LAB^2$ , will be empty.

<sup>5</sup>  $Land^d$  is final factor.

Table 3.2: Intermediate CES nests in production

Activity	Bundle composition
<b>Crops</b>	
VA	$CES (LAB^1, VA^1)$
VA <sup>1</sup>	$CES (ND^2, VA^2)$
VA <sup>2</sup>	$CES (Land^d, KEF)$
<b>Livestock</b>	
VA	$CES (VA^1, VA^2)$
VA <sup>1</sup>	$CES (LAB^1, KEF)$
VA <sup>2</sup>	$CES (Land^d, ND^2)$
<b>Default</b>	
VA	$CES (LAB^1, VA^1)$
VA <sup>1</sup>	$CES (Land^d, KEF)$

The equations for the intermediate nests will be described according to the demand for the individual component bundles, with the price equations described subsequently. The first set of equations determines the bundles  $VA^1$  and  $VA^2$ . The former is a share of  $VA$  for all activities. The latter is a share of  $VA^1$  in the case of crops and of  $VA$  in the case of livestock. The default production structure does not require the  $VA^2$  bundle.

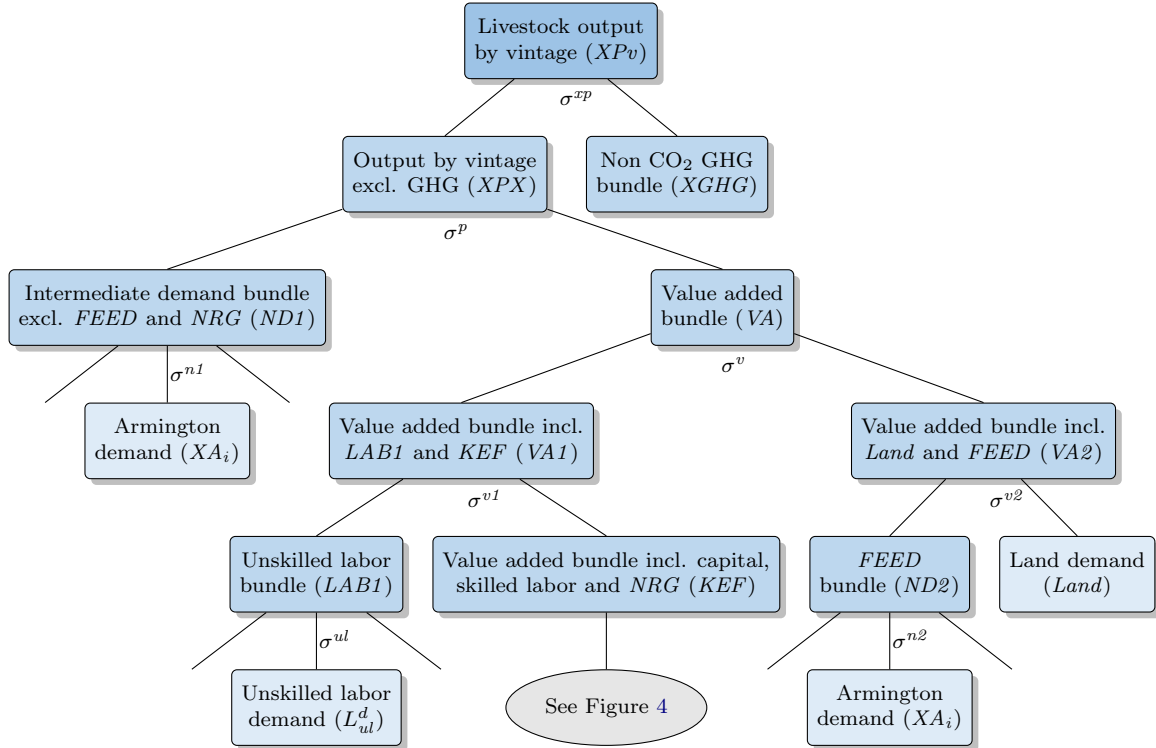


Figure 2: Livestock production nest

$$VA_{r,a,v}^1 = \alpha_{r,a,v}^{va1} \left( \frac{PVA_{r,a,v}}{PVA_{r,a,v}^1} \right)^{\sigma_{r,a,v}^v} VA_{r,a,v} \quad (\text{P-10})$$

$$VA_{r,a,v}^2 = \begin{cases} \alpha_{r,a,v}^{va2} \left( \frac{PVA_{r,a,v}}{PVA_{r,a,v}^2} \right)^{\sigma_{r,a,v}^{v1}} VA_{r,a,v}^1 & \text{if } a \in \{Crops\} \\ \alpha_{r,a,v}^{va2} \left( \frac{PVA_{r,a,v}}{PVA_{r,a,v}^2} \right)^{\sigma_{r,a,v}^v} VA_{r,a,v} & \text{if } a \in \{Livestock\} \end{cases} \quad (\text{P-11})$$

The next set of equations determine the bundles  $LAB^1$ ,  $KEF$  and  $ND^2$ . The subsequent decomposition of these three bundles will be identical for all activities. The  $LAB^1$  bundle is a share of  $VA^1$  for *Livestock* and  $VA$  for all other activities. The  $KEF$  bundle is a share of  $VA^2$  for *Crops* and  $VA^1$  for all other activities. The  $ND^2$  bundle is a share of  $VA^1$  for *Crops* and  $VA^2$  for *Livestock*. The  $ND^2$  bundle is not used in the *Default* production structure.

$$LAB_{r,a}^1 = \begin{cases} \sum_v \alpha_{r,a,v}^{l1} \left( \frac{PVA_{r,a,v}}{PLAB_{r,a}^1} \right)^{\sigma_{r,a,v}^v} VA_{r,a,v} & \text{if } a \in \{Default\} \\ \sum_v \alpha_{r,a,v}^{l1} \left( \frac{PVA_{r,a,v}}{PLAB_{r,a}^1} \right)^{\sigma_{r,a,v}^{v1}} VA_{r,a,v}^1 & \text{if } a \in \{Livestock\} \end{cases} \quad (\text{P-12})$$

$$KEF_{r,a,v} = \begin{cases} \alpha_{r,a,v}^{kef} \left( \frac{PVA_{r,a,v}^2}{PKEF_{r,a,v}} \right)^{\sigma_{r,a,v}^{v2}} VA_{r,a,v}^2 & \text{if } a \in \{Crops\} \\ \alpha_{r,a,v}^{kef} \left( \frac{PVA_{r,a,v}^1}{PKEF_{r,a,v}} \right)^{\sigma_{r,a,v}^{v1}} VA_{r,a,v}^1 & \text{if } a \in \{Default\} \end{cases} \quad (\text{P-13})$$

$$ND_{r,a}^2 = \begin{cases} \sum_v \alpha_{r,a,v}^{nd2} \left( \frac{PVA_{r,a,v}^1}{PND_{r,a}^2} \right)^{\sigma_{r,a,v}^{v1}} VA_{r,a,v}^1 & \text{if } a \in \{Crops\} \\ \sum_v \alpha_{r,a,v}^{nd2} \left( \frac{PVA_{r,a,v}^2}{PND_{r,a}^2} \right)^{\sigma_{r,a,v}^{v2}} VA_{r,a,v}^2 & \text{if } a \in \{Livestock\} \end{cases} \quad (\text{P-14})$$

The final demand equation in this set of intermediate nests determines the demand for the land factor,  $Land^d$ . It is a share of  $VA^2$  for *Crops* and *Livestock* and a share of  $VA^1$  for the *Default* activities. The parameter  $\lambda^t$  allows for efficiency improvement in the use of land. The relevant price of land is  $PLand^p$  that represents the user (or agent) price of land. It is equal to the market price of land adjusted for an activity-specific tax or subsidy.

$$Land_{r,a}^d = \begin{cases} \sum_v \alpha_{r,a,v}^{land} \left( \frac{\lambda_{r,a,v}^t PVA_{r,a,v}^2}{PLand_{r,a}^p} \right)^{\sigma_{r,a,v}^{v2}} \frac{VA_{r,a,v}^2}{\lambda_{r,a,v}^t} & \text{if } a \in \{Crops\} \\ \sum_v \alpha_{r,a,v}^{land} \left( \frac{\lambda_{r,a,v}^t PVA_{r,a,v}^2}{PLand_{r,a}^p} \right)^{\sigma_{r,a,v}^{v2}} \frac{VA_{r,a,v}^2}{\lambda_{r,a,v}^t} & \text{if } a \in \{Livestock\} \\ \sum_v \alpha_{r,a,v}^{land} \left( \frac{\lambda_{r,a,v}^t PVA_{r,a,v}^1}{PLand_{r,a}^p} \right)^{\sigma_{r,a,v}^{v1}} \frac{VA_{r,a,v}^1}{\lambda_{r,a,v}^t} & \text{if } a \in \{Default\} \end{cases} \quad (\text{P-15})$$

The description of the intermediate nests is finished with the determination of the prices of the intermediate bundles  $VA$ ,  $VA^1$  and  $VA^2$ . Equation (P-16) determines the price of the  $VA$  bundle,

PVA. Equation (P-17) determines the price of the  $VA^1$  bundle,  $PVA^1$ . Equation (P-17) determines the price of the  $VA^2$  bundle,  $PVA^2$ .

$$PVA_{r,a,v} = \begin{cases} \left[ \alpha_{r,a,v}^{ll} (PLAB_{r,a}^1)^{1-\sigma_{r,a,v}^v} \right. \\ \quad \left. + \alpha_{r,a,v}^{va1} (PVA_{r,a,v}^1)^{1-\sigma_{r,a,v}^v} \right]^{\frac{1}{1-\sigma_{r,a,v}^v}} & \text{if } a \in \{Default\} \\ \left[ \alpha_{r,a,v}^{va1} (PVA_{r,a,v}^1)^{1-\sigma_{r,a,v}^v} \right. \\ \quad \left. + \alpha_{r,a,v}^{va2} (PVA_{r,a,v}^2)^{1-\sigma_{r,a,v}^v} \right]^{\frac{1}{1-\sigma_{r,a,v}^v}} & \text{if } a \in \{Livestock\} \end{cases} \quad (P-16)$$

$$PVA_{r,a,v}^1 = \begin{cases} \left[ \alpha_{r,a,v}^{nd2} (PND_{r,a}^2)^{1-\sigma_{r,a,v}^{v1}} \right. \\ \quad \left. + \alpha_{r,a,v}^{va2} (PVA_{r,a,v}^2)^{1-\sigma_{r,a,v}^{v1}} \right]^{\frac{1}{1-\sigma_{r,a,v}^{v1}}} & \text{if } a \in \{Crops\} \\ \left[ \alpha_{r,a,v}^{ll} (PLAB_{r,a}^1)^{1-\sigma_{r,a,v}^{v1}} \right. \\ \quad \left. + \alpha_{r,a,v}^{kef} (PKEF_{r,a,v})^{1-\sigma_{r,a,v}^{v1}} \right]^{\frac{1}{1-\sigma_{r,a,v}^{v1}}} & \text{if } a \in \{Livestock\} \\ \left[ \alpha_{r,a,v}^{land} \left( \frac{PLand_{r,a}^p}{\lambda_{r,a,v}^l} \right)^{1-\sigma_{r,a,v}^{v1}} \right. \\ \quad \left. + \alpha_{r,a,v}^{kef} (PKEF_{r,a,v})^{1-\sigma_{r,a,v}^{v1}} \right]^{\frac{1}{1-\sigma_{r,a,v}^{v1}}} & \text{if } a \in \{Default\} \end{cases} \quad (P-17)$$

$$PVA_{r,a,v}^2 = \begin{cases} \left[ \alpha_{r,a,v}^{land} \left( \frac{PLand_{r,a}^p}{\lambda_{r,a,v}^l} \right)^{1-\sigma_{r,a,v}^{v2}} \right. \\ \quad \left. + \alpha_{r,a,v}^{kef} (PKEF_{r,a,v})^{1-\sigma_{r,a,v}^{v2}} \right]^{\frac{1}{1-\sigma_{r,a,v}^{v2}}} & \text{if } a \in \{Crops\} \\ \left[ \alpha_{r,a,v}^{land} \left( \frac{PLand_{r,a}^p}{\lambda_{r,a,v}^l} \right)^{1-\sigma_{r,a,v}^{v2}} \right. \\ \quad \left. + \alpha_{r,a,v}^{nd2} (PND_{r,a}^2)^{1-\sigma_{r,a,v}^{v2}} \right]^{\frac{1}{1-\sigma_{r,a,v}^{v2}}} & \text{if } a \in \{Livestock\} \end{cases} \quad (P-18)$$

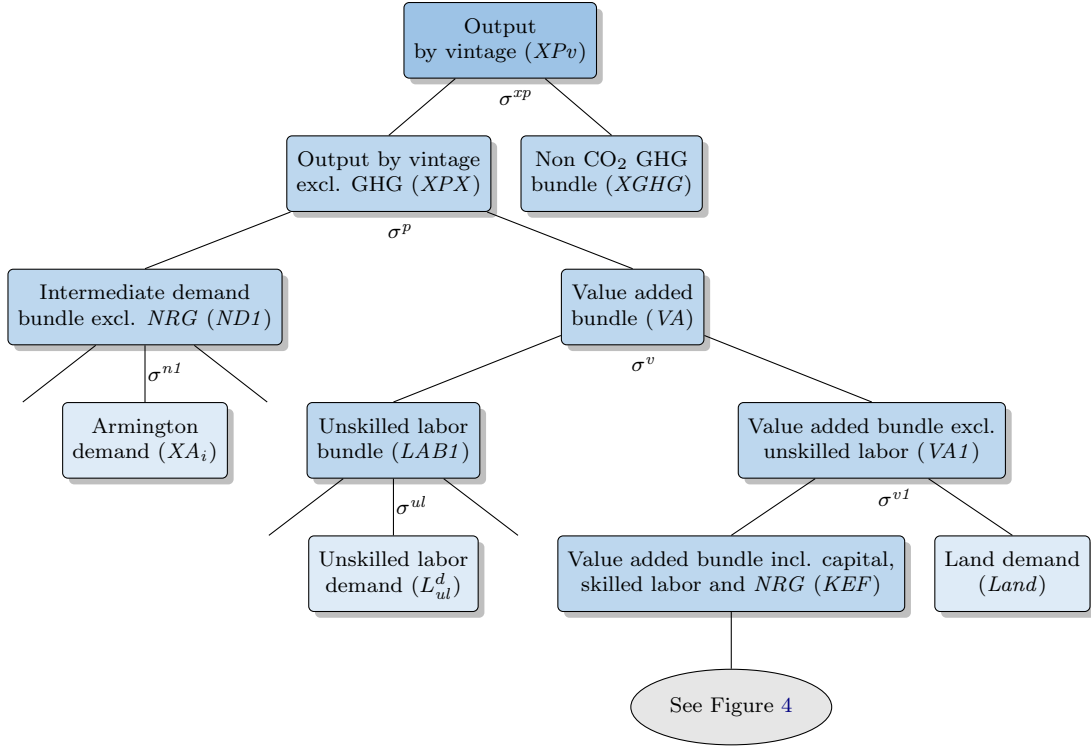
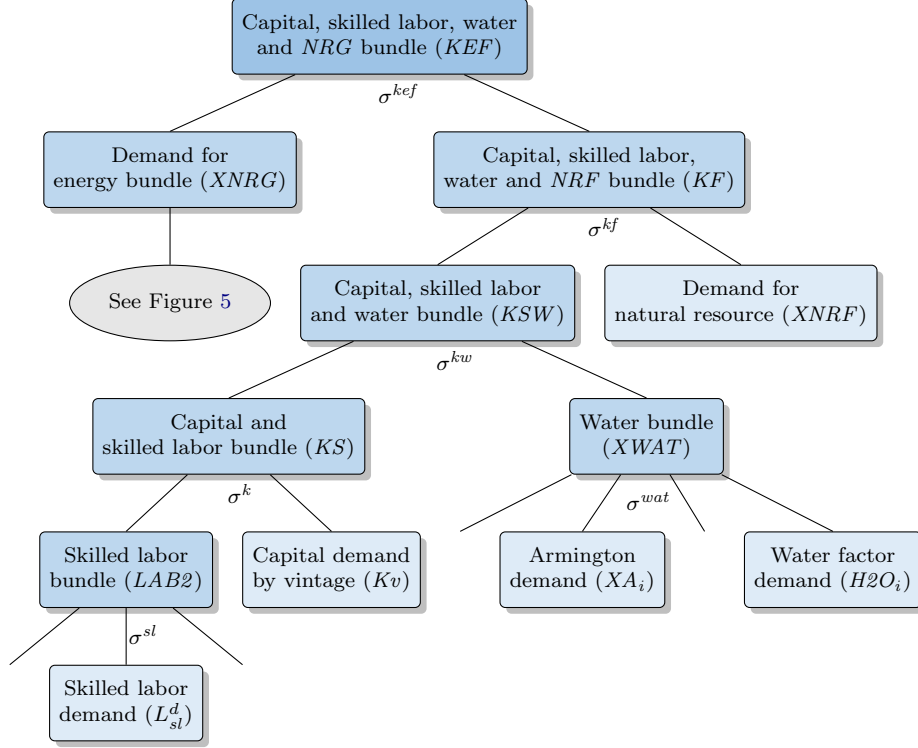


Figure 3: **Default production nest**

The subsequent production nests are identical for all three production prototypes. The *KEF* bundle is composed of the *KF* bundle, a composition of capital, skilled labor, water and natural resources, and the energy bundle, *XNRG*. Equations (P-19) and (P-20) represent the derived demands for the *KF* and *XNRG* bundles respectively, with the bundle prices represented by *PKF* and *PNRG*. The main substitution elasticity is  $\sigma^{kef}$ . Equation (P-21) represents the the CES dual price expression for the price of the *KEF* bundle, *PKEF*.


 Figure 4: **KEF bundle nest**

$$KF_{r,a,v} = \alpha_{r,a,v}^{kf} \left( \frac{PKEF_{r,a,v}}{PKF_{r,a,v}} \right)^{\sigma_{r,a,v}^{kef}} KEF_{r,a,v} \quad (\text{P-19})$$

$$XNRG_{r,a,v} = \alpha_{r,a,v}^e \left( \frac{PKEF_{r,a,v}}{PNRG_{r,a,v}} \right)^{\sigma_{r,a,v}^{kef}} KEF_{r,a,v} \quad (\text{P-20})$$

$$PKEF_{r,a,v} = \left[ \alpha_{r,a,v}^{kf} (PKF_{r,a,v})^{1-\sigma_{r,a,v}^{kef}} + \alpha_{r,a,v}^e (PNRG_{r,a,v})^{1-\sigma_{r,a,v}^{kef}} \right]^{\frac{1}{1-\sigma_{r,a,v}^{kef}}} \quad (\text{P-21})$$

The  $KF$  bundle is composed of the  $KSW$  bundle, a composition of capital, skilled labor and water and the sector-specific natural resource,  $XNRF^d$ . Equations (P-22) and (P-23) represent the derived demands for the  $KSW$  bundle and the sector-specific natural resource,  $XNRF^d$  (when present), respectively, with the respective prices represented by  $PKSW$  and  $PNRF^p$ . The main substitution elasticity is  $\sigma^{kf}$ . The natural resource factor has an efficiency factor,  $\lambda^{nrf}$ , that is typically exogenous. Equation (P-24) represents the the CES dual price expression for the price of the  $KF$  bundle,  $PKF$ .

$$KSW_{r,a,v} = \alpha_{r,a,v}^{ksw} \left( \frac{PKF_{r,a,v}}{PKSW_{r,a,v}} \right)^{\sigma_{r,a,v}^{kf}} KF_{r,a,v} \quad (\text{P-22})$$

$$XNRF_{r,a}^d = \sum_v \alpha_{r,a,v}^{nrf} \left( \frac{\lambda_{r,a,v}^{nrf} PKF_{r,a,v}}{PNRF_{r,a}^p} \right)^{\sigma_{r,a,v}^{kf}} \frac{KF_{r,a,v}}{\lambda_{r,a,v}^{nrf}} \quad (\text{P-23})$$

$$PKF_{r,a,v} = \left[ \alpha_{r,a,v}^{ksw} (PKSW_{r,a,v})^{1-\sigma_{r,a,v}^{kf}} + \alpha_{r,a,v}^{nrf} \left( \frac{PNRF_{r,a}}{\lambda_{r,a,v}^{nrf}} \right)^{1-\sigma_{r,a,v}^{kf}} \right]^{\frac{1}{1-\sigma_{r,a,v}^{kf}}} \quad (\text{P-24})$$

The  $KSW$  bundle is composed of the  $KS$  bundle, a composition of capital and skilled labor and the water bundle,  $XWAT$ . Equations (P-25) and (P-26) represent the derived demands for the  $KS$  bundle and the water bundle, respectively, with the respective prices represented by  $PKS$  and  $PWAT$ . The main substitution elasticity is  $\sigma^{kw}$ . Equation (P-27) represents the CES dual price expression for the price of the  $KSW$  bundle,  $PKSW$ .

$$KS_{r,a,v} = \alpha_{r,a,v}^{ks} \left( \frac{PKSW_{r,a,v}}{PKS_{r,a,v}} \right)^{\sigma_{r,a,v}^{kw}} KSW_{r,a,v} \quad (\text{P-25})$$

$$XWAT_{r,a} = \sum_v \alpha_{r,a,v}^{wat} \left( \frac{PKSW_{r,a,v}}{PWAT_{r,a}} \right)^{\sigma_{r,a,v}^{kw}} KSW_{r,a,v} \quad (\text{P-26})$$

$$PKSW_{r,a,v} = \left[ \alpha_{r,a,v}^{ks} (PKS_{r,a,v})^{1-\sigma_{r,a,v}^{kw}} + \alpha_{r,a,v}^{wat} (PWAT_{r,a})^{1-\sigma_{r,a,v}^{kw}} \right]^{\frac{1}{1-\sigma_{r,a,v}^{kw}}} \quad (\text{P-27})$$

The  $KS$  bundle is composed of capital (by vintage),  $Kv$ , and the skilled labor bundle  $LAB^2$ . Equations (P-28) and (P-29) represent the derived demands for capital and the skilled labor bundle, respectively, with the respective prices represented by  $PK^p$  and  $PLAB^2$ . The main substitution elasticity is  $\sigma^k$ . The capital factor has an efficiency factor,  $\lambda^k$ , that is typically exogenous. Equation (P-30) represents the the CES dual price expression for the price of the  $KS$  bundle,  $PKS$ .

$$K_{r,a,v}^v = \alpha_{r,a,v}^k \left( \frac{\lambda_{r,a,v}^k PKS_{r,a,v}}{PK_{r,a,v}^p} \right)^{\sigma_{r,a,v}^k} \frac{KS_{r,a,v}}{\lambda_{r,a,v}^k} \quad (\text{P-28})$$

$$LAB_{r,a}^2 = \sum_v \alpha_{r,a,v}^{l2} \left( \frac{PKS_{r,a,v}}{PLAB_{r,a}^2} \right)^{\sigma_{r,a,v}^k} KS_{r,a,v} \quad (\text{P-29})$$

$$PKS_{r,a,v} = \left[ \alpha_{r,a,v}^k \left( \frac{PK_{r,a,v}^p}{\lambda_{r,a,v}^k} \right)^{1-\sigma_{r,a,v}^k} + \alpha_{r,a,v}^{l2} (PLAB_{r,a}^2)^{1-\sigma_{r,a,v}^k} \right]^{\frac{1}{1-\sigma_{r,a,v}^k}} \quad (\text{P-30})$$

The next set of CES nests decomposes the two labor bundles, unskilled and skilled labor, respectively given by  $LAB^1$  and  $LAB^2$ . Equation (P-31) provides the decomposition of the bundles where the key substitution elasticities are  $\sigma^{ul}$  and  $\sigma^{sl}$ , the producer cost of labor is given by  $W^p$ , and  $\lambda^l$  represents labor efficiency. Equation (P-32) determines the price of the unskilled labor bundle,  $PLAB^1$ , and equation (P-33) determines the price of the skilled labor bundle,  $PLAB^2$ . The user determines the composition of the labor bundles by mapping the specific skill types to either the unskilled or skilled labor bundles. All skill levels can be mapped to one of the two bundles, in which case the other bundle would be empty and not included in the model definition.

$$L_{r,l,a}^d = \begin{cases} \alpha_{r,l,a}^l \left( \frac{\lambda_{r,l,a}^l PLAB_{r,a}^1}{W_{r,l,a}^p} \right)^{\sigma_{r,a}^{ul}} \frac{LAB_{r,a}^1}{\lambda_{r,l,a}^l} & \text{if } l \in \{Unskilled\} \\ \alpha_{r,l,a}^l \left( \frac{\lambda_{r,l,a}^l PLAB_{r,a}^2}{W_{r,l,a}^p} \right)^{\sigma_{r,a}^{sl}} \frac{LAB_{r,a}^2}{\lambda_{r,l,a}^l} & \text{if } l \in \{Skilled\} \end{cases} \quad (\text{P-31})$$

$$PLAB_{r,a}^1 = \left[ \sum_{ul} \alpha_{r,ul,a}^l \left( \frac{W_{r,ul,a}^p}{\lambda_{r,ul,a}^l} \right)^{1-\sigma_{r,a}^{ul}} \right]^{\frac{1}{1-\sigma_{r,a}^{ul}}} \quad (\text{P-32})$$

$$PLAB_{r,a}^2 = \left[ \sum_{sl} \alpha_{r,sl,a}^l \left( \frac{W_{r,sl,a}^p}{\lambda_{r,sl,a}^l} \right)^{1-\sigma_{r,a}^{sl}} \right]^{\frac{1}{1-\sigma_{r,a}^{sl}}} \quad (\text{P-33})$$

Similar to the labor bundles, the next set of CES nests decomposes the three intermediate demand bundles,  $ND^1$ ,  $ND^2$  and  $XWAT$ . Recall that  $ND^2$  contains activity-specific inputs such as fertilizers in crops and feed in livestock. The  $ND^1$  bundle contains all of the other intermediate goods—with the exception of water and energy goods. The  $XWAT$  bundle contains all designated water commodities from intermediate demand as well as the water factor in some sectors—such as irrigated agriculture. Equation (P-34) provides the decomposition of the bundles where the key substitution elasticities are  $\sigma^{nd1}$ ,  $\sigma^{nd2}$  and  $\sigma^{wat}$ , the producer cost of intermediate goods is given by  $PA^a$ , and  $\lambda^{io}$  represents an efficiency factor for the use of intermediate goods. Equation (P-35) determines the price of the  $ND^1$  bundle,  $PND^1$  bundle, and equation (P-36) determines the price of the  $ND^2$  bundle,  $PND^2$ . The user determines the composition of the intermediate demand bundles by mapping the specific intermediate commodities to one of the two bundles.<sup>6</sup>

$$XA_{r,i,a} = \begin{cases} \alpha_{r,i,a}^{io} \left( \frac{\lambda_{r,i,a}^{io} PND_{r,a}^1}{PA_{r,i,a}^a} \right)^{\sigma_{r,a}^{nd1}} \frac{ND_{r,a}^1}{\lambda_{r,i,a}^{io}} & \text{if } i \in \{ND1\} \\ \alpha_{r,i,a}^{io} \left( \frac{\lambda_{r,i,a}^{io} PND_{r,a}^2}{PA_{r,i,a}^a} \right)^{\sigma_{r,a}^{nd2}} \frac{ND_{r,a}^2}{\lambda_{r,i,a}^{io}} & \text{if } i \in \{ND2\} \\ \alpha_{r,i,a}^{io} \left( \frac{\lambda_{r,i,a}^{io} PWAT_{r,a}}{PA_{r,i,a}^a} \right)^{\sigma_{r,a}^{wat}} \frac{XWAT_{r,a}}{\lambda_{r,i,a}^{io}} & \text{if } i \in \{iw\} \end{cases} \quad (\text{P-34})$$

$$PND_{r,a}^1 = \left[ \sum_{i \in \{ND1\}} \alpha_{r,i,a}^{io} \left( \frac{PA_{r,i,a}^a}{\lambda_{r,i,a}^{io}} \right)^{1-\sigma_{r,a}^{nd1}} \right]^{\frac{1}{1-\sigma_{r,a}^{nd1}}} \quad (\text{P-35})$$

$$PND_{r,a}^2 = \left[ \sum_{i \in \{ND2\}} \alpha_{r,i,a}^{io} \left( \frac{PA_{r,i,a}^a}{\lambda_{r,i,a}^{io}} \right)^{1-\sigma_{r,a}^{nd2}} \right]^{\frac{1}{1-\sigma_{r,a}^{nd2}}} \quad (\text{P-36})$$

Equation (P-37) determines the demand for the water factor.<sup>7</sup> Equation (P-38) determines the price of the  $XWAT$  bundle, where the subset  $iw$  spans the set of water commodities.<sup>8</sup>

$$H2O_{r,a}^d = \alpha_{r,a}^{h2o} \left( \frac{\lambda_{r,a}^{h2o} PWAT_{r,a}}{PH2O_{r,a}^p} \right)^{\sigma_{r,a}^{wat}} \frac{XWAT_{r,a}}{\lambda_{r,a}^{h2o}} \quad (\text{P-37})$$

<sup>6</sup> In the GAMS code the two mappings are driven by the sets `mapi1` and `mapi2`.

<sup>7</sup> At the moment, only irrigated crops have any water demand.

<sup>8</sup> In a standard aggregation of the GTAP database, the subset  $iw$  would be composed of the GTAP commodity labeled `wtr`.



$$PWAT_{r,a} = \left[ \sum_{i \in \{iw\}} \alpha_{r,i,a}^{io} \left( \frac{PA_{r,i,a}^a}{\lambda_{r,i,a}^{io}} \right)^{1-\sigma_{r,a}^{wat}} + \alpha_{r,a}^{h2o} \left( \frac{PH2O_{r,a}^p}{\lambda_{r,a}^{h2o}} \right)^{1-\sigma_{r,a}^{wat}} \right]^{\frac{1}{1-\sigma_{r,a}^{wat}}} \quad (\text{P-38})$$

The final set of nests in production concern the energy bundle,  $XNRG$ . It will be decomposed into demand for the energy commodities, figure 5. The energy bundle is first decomposed into electric and non-electric bundles. The latter is then decomposed into a coal bundle and a non-coal bundle (or the oil & gas bundle). The oil & gas bundle is then split into a gas bundle and an oil bundle. The four remaining bundles—electric, coal, oil and gas—represent a combination of existing or future energy sources. In the case of the original GTAP database, the electric bundle would hold the 'ely' commodity, the coal bundle would hold the 'coa' commodity, the oil bundle would hold the 'oil' and 'p-c' commodities and the gas bundle would hold the 'gas' and 'gdt' commodities. Non-GTAP commodities would be mapped to one of the existing bundles. For example, 'p-c' could be split into gasoline and diesel, and/or could include ethanol or bio-diesel.

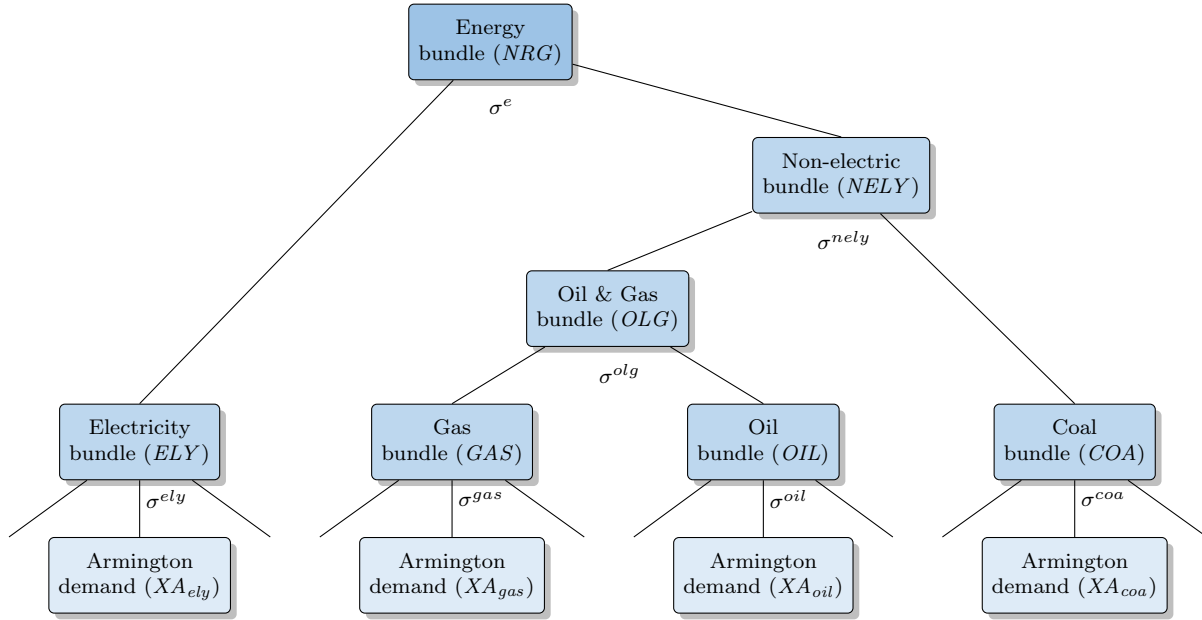


Figure 5: **Energy bundle nest**

Equation (P-39) determines the demand for the electric bundle,  $XA^{ely}$ .<sup>9</sup> Equation (P-40) determines the demand for the non-electric bundle,  $XNELY$ . In both equations, the key substitution elasticity is given by  $\sigma^e$ . Equation (P-41) then describes the aggregate price of energy,  $PNRG$ .

$$XA_{r,a,v}^{ely} = \alpha_{r,a,v}^{ely} \left( \frac{PNRG_{r,a,v}}{PA_{r,a,v}^{ely}} \right)^{\sigma_{r,a,v}^e} XNRG_{r,a,v} \quad (\text{P-39})$$

$$XNELY_{r,a,v} = \alpha_{r,a,v}^{nely} \left( \frac{PNRG_{r,a,v}}{PNELY_{r,a,v}} \right)^{\sigma_{r,a,v}^e} XNRG_{r,a,v} \quad (\text{P-40})$$

<sup>9</sup> In the GAMS code, the demand for the four energy bundles ('ely', 'gas', 'oil' and 'coa') are represented by the variable  $\mathbf{xNRG}$  that has an additional dimension representing the four main energy carriers. This simplifies the decomposition of these bundles that only requires one set of equations rather than four. In principle, it also makes it somewhat easier to increase the number of energy bundles.

$$PNRG_{r,a,v} = \left[ \alpha_{r,a,v}^{ely} \left( PA_{r,a,v}^{ely} \right)^{1-\sigma_{r,a,v}^e} + \alpha_{r,a,v}^{nely} \left( PNELY_{r,a,v} \right)^{1-\sigma_{r,a,v}^e} \right]^{\frac{1}{1-\sigma_{r,a,v}^e}} \quad (\text{P-41})$$

Equation (P-42) determines the demand for the coal bundle,  $XA^{coa}$ . Equation (P-43) determines the demand for the oil & gas bundle,  $XOLG$ . In both equations, the key substitution elasticity is given by  $\sigma^{nely}$ . Equation (P-44) then describes the aggregate price of the non-electric bundle,  $PNELY$ .

$$XA_{r,a,v}^{coa} = \alpha_{r,a,v}^{coa} \left( \frac{PNELY_{r,a,v}}{PA_{r,a,v}^{coa}} \right)^{\sigma_{r,a,v}^{nely}} XNELY_{r,a,v} \quad (\text{P-42})$$

$$XOLG_{r,a,v} = \alpha_{r,a,v}^{olg} \left( \frac{PNELY_{r,a,v}}{POLG_{r,a,v}} \right)^{\sigma_{r,a,v}^{nely}} XNELY_{r,a,v} \quad (\text{P-43})$$

$$PNELY_{r,a,v} = \left[ \alpha_{r,a,v}^{coa} \left( PA_{r,a,v}^{coa} \right)^{1-\sigma_{r,a,v}^{nely}} + \alpha_{r,a,v}^{olg} \left( POLG_{r,a,v} \right)^{1-\sigma_{r,a,v}^{nely}} \right]^{\frac{1}{1-\sigma_{r,a,v}^{nely}}} \quad (\text{P-44})$$

The remaining two energy bundles are oil and gas and emanate from the  $XOLG$  bundle. Equation (P-45) determines the demand for the oil bundle,  $XA^{oil}$ . Equation (P-46) determines the demand for the gas bundle,  $XA^{gas}$ . In both equations, the key substitution elasticity is given by  $\sigma^{OLG}$ . Equation (P-47) then describes the aggregate price of the oil & gas bundle,  $POLG$ .

$$XA_{r,a,v}^{oil} = \alpha_{r,a,v}^{oil} \left( \frac{POLG_{r,a,v}}{PA_{r,a,v}^{oil}} \right)^{\sigma_{r,a,v}^{olg}} XOLG_{r,a,v} \quad (\text{P-45})$$

$$XA_{r,a,v}^{gas} = \alpha_{r,a,v}^{gas} \left( \frac{POLG_{r,a,v}}{PA_{r,a,v}^{gas}} \right)^{\sigma_{r,a,v}^{olg}} XOLG_{r,a,v} \quad (\text{P-46})$$

$$POLG_{r,a,v} = \left[ \alpha_{r,a,v}^{oil} \left( PA_{r,a,v}^{oil} \right)^{1-\sigma_{r,a,v}^{olg}} + \alpha_{r,a,v}^{gas} \left( PA_{r,a,v}^{gas} \right)^{1-\sigma_{r,a,v}^{olg}} \right]^{\frac{1}{1-\sigma_{r,a,v}^{olg}}} \quad (\text{P-47})$$

The final nest in the energy bundle is to decompose the four aggregate energy bundles into their constituent parts that represent the Armington demand for the energy commodities. Equation (P-48) reflects the Armington demand for energy commodity  $e$ ,  $XA$ , where the cost to producers is given by  $PA^a$ . The key substitution elasticity for each energy bundle is given by  $\sigma^{NRG}$ . Equation (P-49) represents the price of the aggregate energy bundles,  $PA^{NRG}$ .

$$XA_{r,e,a} = \sum_v \alpha_{r,e,a,v}^{eio} \left( \frac{\lambda_{r,e,a,v}^e PA_{r,a,v}^{NRG}}{PA_{r,e,a}^a} \right)^{\sigma_{r,a,v}^{NRG}} \frac{XA_{r,a,v}^{NRG}}{\lambda_{r,e,a,v}^e} \quad \text{if } e \in \{NRG\} \quad (\text{P-48})$$

$$PA_{r,a,v}^{NRG} = \left[ \sum_{e \in \{NRG\}} \alpha_{r,e,a,v}^{eio} \left( \frac{PA_{r,e,a}^a}{\lambda_{r,e,a,v}^e} \right)^{1-\sigma_{r,a,v}^{NRG}} \right]^{\frac{1}{1-\sigma_{r,a,v}^{NRG}}} \quad (\text{P-49})$$

### 3.3 Commodity supply

The model allows for each activity  $a$  to produce one or more commodities (indexed by  $i$ ). For example, the ethanol industry could produce both ethanol as well as distiller's dried grain with solubles (or DDGS). Similarly, a single commodity can be produced by one or more activities. For example, the electricity commodity can be produced by multiple generation activities—thermal, nuclear, hydro, renewables, etc. Joint production is captured by a CET transformation function, with the possibility of perfect transformation. The aggregation of output from multiple activities is captured with a CES preference function, again with the possibility of commodity homogeneity, in which case the law-of-one-price holds.<sup>10</sup>

As an option to the standard model, power generation can be treated in a special fashion compared to other goods and services. The aggregation of production across activities is normally determined using a single CES nest. For the power activities, aggregation to the single electricity commodity uses a nested CES structure that is explained in the subsequent section.

#### 3.3.1 Non-electric goods

This section describes the make matrix for all commodities with the exception of electricity that is described subsequently. The equations thus hold for all commodities indexed by  $i$  except for the electricity commodity. The allocation of output,  $XP_{r,a}$ , from activity  $a$ , to supply one or more commodity  $i$ , is described in equation (S-1) that is the standard CET supply allocation expression. The variable  $X$  represents the supply of commodity  $i$  by activity  $a$ . The transformation elasticity is given by  $\omega^s$ . The model allows for perfect transformation, in which case the law-of-one-price holds. Equation (S-2) is in essence an equilibrium condition that determines the aggregate output of activity  $a$ ,  $XP$ . In the case of perfect transformation, it simply aggregates the individual supplies.

$$\begin{cases} X_{r,a,i} = \gamma_{r,a,i}^p \left( \frac{P_{r,a,i}}{PP_{r,a}} \right)^{\omega_{r,a}^s} XP_{r,a} & \text{if } \omega_{r,a}^s \neq \infty \\ P_{r,a,i} = PP_{r,a} & \text{if } \omega_{r,a}^s = \infty \end{cases} \quad (\text{S-1})$$

$$PP_{r,a}XP_{r,a} = \sum_i P_{r,a,i}X_{r,a,i} \quad (\text{S-2})$$

The supply of commodity  $i$  is the (CES) aggregation of output of one or more activities  $a$ . A CES preference function is used for the aggregation. Equation (S-3) determines the 'demand' for output  $a$  to compose commodity  $i$ ,  $X$ .<sup>11</sup> The substitution elasticity is given by  $\sigma^s$ . The model allows for perfect substitution, in which case the law-of-one-price holds. Equation (S-4) determines the (market) price of commodity  $i$ ,  $PS$ .

$$\begin{cases} X_{r,a,i} = \alpha_{r,a,i}^s \left( \frac{PS_{r,i}}{P_{r,a,i}} \right)^{\sigma_{r,i}^s} XS_{r,i} & \text{if } \sigma_{r,i}^s \neq \infty \\ P_{r,a,i} = PS_{r,i} & \text{if } \sigma_{r,i}^s = \infty \end{cases} \quad (\text{S-3})$$

<sup>10</sup> In the standard GTAP database, there is a one-to-one correspondence between activities and commodities, i.e. the make matrix is diagonal. The aggregation facility for the ENVISAGE model allows the user to determine the mapping from activities to commodities. For example, it is possible to have a single agricultural production function that produces a number of agricultural commodities. Similarly, it is possible to have a number of agricultural production activities that produce a single agricultural commodity.

<sup>11</sup> Equation (S-1) determines supply and equation (S-3) determines demand. The supply/demand equilibrium condition is substituted out.

$$PS_{r,i}XS_{r,i} = \sum_a P_{r,a,i}X_{r,a,i} \quad (\text{S-4})$$

### 3.3.2 Domestic supply of electricity

The bundling of electricity uses a nested CES structure instead of a single nest, see figure 6. The top nest combines aggregate power supply with distribution and transmission services to form aggregate domestic electric supply. The power nest combines a number of different (and user-determined) power bundles. Subsequently, each of these power bundles are formed by the different power activities that are segmented into the different power bundles (under user-mapping). For example, the power bundles may be composed of coal-, oil- and gas-generation, nuclear, and all other. Using the GTAP power database, base load coal could be mapped to the coal power bundle, base and peak load oil could be mapped to the oil power bundle, base and peak load gas could be mapped to the gas power bundle, base load nuclear would be mapped to the nuclear power bundle, and all other power activities (wind, solar, hydro and other) could be mapped to the other power bundle. The strategy for future technologies would be to bundle them in power bundles. For example coal capture and storage could be in the coal power bundle, and advanced nuclear could be incorporated in the nuclear power bundle.

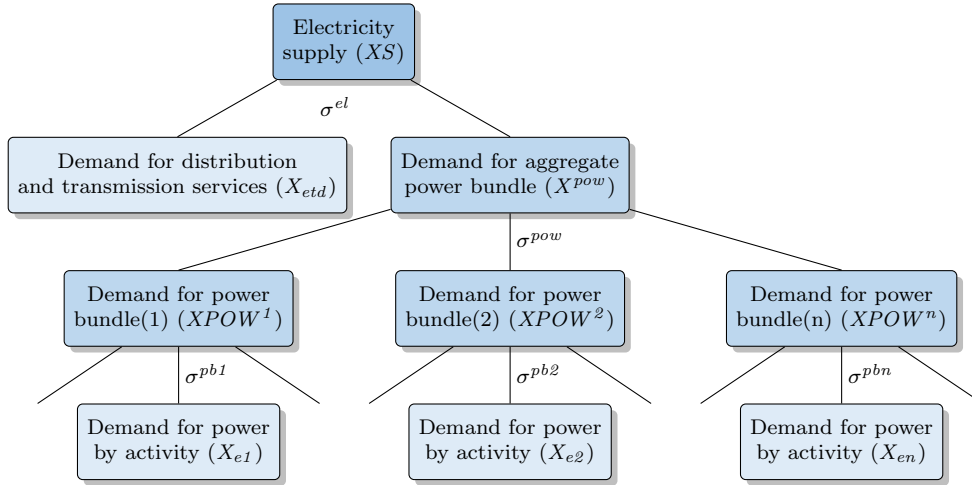


Figure 6: CES nest for power bundle

Equation (S-5) determines the demand for electricity services—indexed by activities  $etd$ —used to produce one or more electric commodities—indexed by  $ely$ .<sup>12</sup> It is linked to the total supply of power,  $XS^{pow}$ , in a CES bundle. The normal specification assumes a Leontief technology, i.e. a substitution elasticity of zero. Equation (S-6) determines the demand for the power bundle, it is a bundle of all electricity generation, and excludes the transmission and distribution services. Equation (S-7) determines the supply price of aggregate electricity.

$$X_{r,etd,ely} = \alpha_{r,etd,ely}^s \left( \frac{PS_{r,ely}}{P_{r,etd,ely}} \right)^{\sigma_{r,ely}^{el}} XS_{r,ely} \quad (\text{S-5})$$

$$XPOW_{r,ely} = \alpha_{r,ely}^{pow} \left( \frac{PS_{r,ely}}{PPOW_{r,ely}} \right)^{\sigma_{r,ely}^{el}} XS_{r,ely} \quad (\text{S-6})$$

<sup>12</sup> Typically there is a single transmission and distribution activity and a single electricity commodity.

$$PS_{r,ely} = \left[ \alpha_{r,etd,ely}^s (P_{r,etd,ely})^{1-\sigma_{r,ely}^{el}} + \alpha_{r,ely}^{pow} (PPOW_{r,ely})^{1-\sigma_{r,ely}^{el}} \right]^{1/(1-\sigma_{r,ely}^{el})} \quad (\text{S-7})$$

The following stage decomposes aggregate demand for power into a user-determined number of power bundles—indexed by  $pb$ . Equation (S-8) determines the demand for the power bundles. Notice that the aggregate price used in the expression is  $PPOWN$  and not  $PPOW$ . The latter is the aggregate, or average price of the power bundle. The former is a price index that is defined in equation (S-9). In the standard CES, the two price concepts are identical. The power decomposition uses the so-called adjusted CES, which preserves the additivity of the CES components (see Appendix A). The demand expressions in both versions of the CES are similar. However, the expression for the aggregate price index differs and the price index is not equal to the average price (as calculated using the zero profit condition). Thus, equation (S-10) is added that evaluates the average price,  $PPOW$ .

$$XPB_{r,pb,ely} = \alpha_{r,pb,ely}^{pb} \left( \frac{PPOWN_{r,ely}}{PPB_{r,pb,ely}} \right)^{\sigma_{r,ely}^{pow}} XPOW_{r,ely} \quad (\text{S-8})$$

$$PPOWN_{r,ely} = \left[ \sum_{pb} \alpha_{r,pb,ely}^{pb} (PPB_{r,pb,ely})^{-\sigma_{r,ely}^{pow}} \right]^{-1/\sigma_{r,ely}^{pow}} \quad (\text{S-9})$$

$$PPOW_{r,ely} XPOW_{r,ely} = \sum_{pb} PPB_{r,pb,ely} XPB_{r,pb,ely} \quad (\text{S-10})$$

The subsequent nest decomposes the the various power bundles into component power activities. Each power activity is mapped to one of the aggregate power bundles. Equation (S-11) determines the demand for power generated by power activity  $elya$  that is mapped to power bundle  $pb$ . Equations (S-12) and (S-13) determine respectively the price index for the power bundle  $pb$ , as derived from the adjusted CES price index expression, and the average price of the power bundle  $pb$  using the zero profit condition.

$$X_{r,elya,ely} = \alpha_{r,elya,ely}^s \left( \frac{PPBN_{r,pb,ely}}{P_{r,elya,ely}} \right)^{\sigma_{r,pb,ely}^{pb}} XPB_{r,pb,ely} \quad \text{if } elya \in pb \quad (\text{S-11})$$

$$PPBN_{r,pb,ely} = \left[ \sum_{elya \in pb} \alpha_{r,elya,ely}^s (P_{r,elya,ely})^{-\sigma_{r,pb,ely}^{pb}} \right]^{-1/\sigma_{r,pb,ely}^{pb}} \quad (\text{S-12})$$

$$PPB_{r,pb,ely} XPB_{r,pb,ely} = \sum_{elya \in pb} P_{r,elya,ely} X_{r,elya,ely} \quad (\text{S-13})$$

### 3.4 Income block

There are three domestic final demand agents—households ( $h$ ), an aggregate government sector ( $gov$ ) and an aggregate investment sector ( $inv$ ). All factor income, net of taxes, accrues to the private household, government revenues are generated by the various indirect taxes in the economy as well as direct taxes, and investment income is equated to the sum of domestic and foreign savings. A portion of capital income flows to a 'global' holder of equity that then portions out profits from the global fund. Remittances are also incorporated and are fully bilateral.

The depreciation allowance is calculated as the replacement cost of the estimated depreciation, equation Y-1. The parameter  $\delta^f$  is allowed to differ from the physical rate of depreciation,  $\delta$ , though in most cases it will be identical. The variable  $PF D_{inv}$  is the unit cost of investment and  $K^s$  is the non-normalized level of the aggregate capital stock. The normalized level of the capital stock, further described below, is scaled to the initial aggregate remuneration of capital, i.e. it's price in the base year is 1. The non-normalized level is needed for calculating the depreciation allowance and in the dynamic equation for updating the aggregate capital stock.

$$DeprY_r = \delta_r^f PF D_{r,inv} K_r^s \quad (\text{Y-1})$$

The model incorporates some level of income flow from labor and income. A portion of each region's profits (net of taxes) flows to a global equity fund that disburses its aggregate income across regions. Equation Y-2 represents the flow of a region's profits net of taxes,  $YQTF$ , to the global equity fund. Total income for the global equity fund,  $TrustY$ , is provided by equation Y-3. Foreign profit inflows,  $YQHT$ , are represented by equation Y-4. Equation Y-5 determines remittances from country  $r$  to country  $s$  for labor of skill  $l$ .<sup>13</sup> Remittances are calculated net of taxes on wages.

$$YQTF_r = \chi_r^f (1 - \kappa_r^k) \left( \sum_a \left[ \sum_v PK_{r,a,v} K_{r,a,v}^v + \pi_{r,a}^m XP_{r,a} \right] - DeprY_r \right) \quad (\text{Y-2})$$

$$TrustY = \sum_r YQTF_r \quad (\text{Y-3})$$

$$YQHT_r = \chi_r^h TrustY \quad (\text{Y-4})$$

$$Remit_{s,l,r} = \chi_{s,l,r}^r (1 - \kappa_{r,l}^l) \sum_a W_{r,l,a} L_{r,l,a}^d \quad (\text{Y-5})$$

Equation (Y-6) describes household income,  $YH$ . It is the sum across all activities of factor income, at market prices and net of taxes and depreciation.<sup>14</sup> Household income also includes net foreign capital income and net remittances. Factor returns at the price producers pay have a superscript  $p$ , which is absent from the factor returns at market prices. Equation (Y-7) describes disposable income,  $YD$ , where  $\kappa^h$  is the marginal (and average) rate of tax on household income. Macro closure is discussed below.

$$\begin{aligned} YH_r = & \sum_l (1 - \kappa_{r,l}^l) \sum_a W_{r,l,a} L_{r,l,a}^d \\ & + (1 - \kappa_r^k) \left( \sum_a \left[ \sum_v PK_{r,a,v} K_{r,a,v}^v + \pi_{r,a}^m XP_{r,a} \right] - DeprY_r \right) \\ & + (1 - \kappa_r^t) \sum_a PLand_{r,a} Land_{r,a}^d \\ & + (1 - \kappa_r^n) \sum_a PNR F_{r,a} XNR F_{r,a}^d \\ & + (1 - \kappa_r^w) \sum_a PH2O_{r,a} H2O_{r,a}^d \\ & + YQHT_r - YQTF_r + \sum_d \sum_l Remit_{r,l,d} - \sum_s \sum_l Remit_{s,l,r} \end{aligned} \quad (\text{Y-6})$$

<sup>13</sup> In the GMig model, remittances are linked to the labor income of migrants by region of origin, i.e.  $s$ . Herein, the remittances are linked to the entire wage bill (by skill), not the wage bill of migrants from region  $s$ .

<sup>14</sup> Note that taxes on profits are evaluated net of the depreciation allowance.

$$YD_r = \left(1 - \kappa_r^h\right) YH_r \quad (\text{Y-7})$$

The next set of equations describe government revenues, contained in the variable  $YGOV$ , which has an additional index for the different revenue streams ( $gy$ ). Equation (Y-8) describes revenues from production and cost taxes. The production tax is applied on the producer price, inclusive of the markup. The revenue index is given by  $ptx$ . Equation (Y-9) describes revenues generated on the factors of production—labor, capital, land and natural resources. The relevant factor prices are the market prices, i.e. prices received by the factors themselves. The revenue index is given by  $vtx$ . Equation (Y-10) describes revenues generated by consumption of goods, essentially a sales tax. The sum is over all domestic agents (indexed by  $aa$ ). The relevant price is the market price of good  $i$ . The equation incorporates the two different Armington options. In the first case, the sourcing of goods is made at the national (aggregate) level and thus all users face a common Armington price,  $PAT$ , that is then adjusted by the end-user sales tax.<sup>15</sup> The second option assumes a top level Armington sourcing by agent in which case the domestic sales tax is differentiated by source. The revenue index is given by  $itx$ . Equation (Y-11) describes revenues generated by import tariffs. These are summed over all source countries ( $s$ ), where the first regional index is always the exporting region and the second regional index is always the destination (or importing) region. The tariffs are applied to the border (or CIF) price of imports,  $PWM$ .<sup>16</sup> The revenue index is given by  $mtx$ . Equation (Y-12) describes revenues generated by export taxes and subsidies. These are summed over all destination countries ( $d$ ). They are applied to the producer price of exports,  $PE$ . The revenue index is given by  $etx$ . Equation (Y-13) describes revenues generated by carbon taxes. This equation holds for either Armington specification. Details on the carbon tax are described further below. The revenue index is given by  $ctx$ . Finally, direct taxes are described in equation (Y-14), where the revenue index is given by  $dtx$ . Direct taxes are imposed on specific factor incomes and there is a net direct tax (that can be negative) on total household income after factor taxes that balances the government account (for a given deficit level).

$$YGOV_{r,ptx} = \sum_a \left[ \tau_{r,a}^p (PX_{r,a} + \pi_{r,a}^m) XP_{r,a} + \sum_v \tau_{r,a,v}^{uc} UC_{r,a,v} XPv_{r,a,v} \right] \quad (\text{Y-8})$$

$$\begin{aligned} YGOV_{r,vtx} &= \sum_a \left[ \sum_l \tau_{r,l,a}^l W_{r,l,a} L_{r,l,a}^d + \sum_v \tau_{r,a,v}^k PK_{r,a,v} K_{r,a,v}^v \right] \\ &+ \sum_a \left[ \tau_{r,a}^t PLand_{r,a} Land_{r,a}^d + \tau_{r,a}^n PNR_{r,a} XNR_{r,a}^d \right] \\ &+ \sum_a \left[ \tau_{r,a}^w PH2O_{r,a} H2O_{r,a}^d \right] \end{aligned} \quad (\text{Y-9})$$

$$YGOV_{r,itx} = \begin{cases} \sum_{aa} \sum_i \tau_{r,i,aa}^a \gamma_{r,i,aa}^{eda} PAT_{r,i} XA_{r,i,aa} & \text{if } ArmFlag = 0 \\ \sum_{aa} \sum_i \tau_{r,i,aa}^d \gamma_{r,i,aa}^{edd} PDT_{r,i} XD_{r,i,aa} \\ \quad + \tau_{r,i,aa}^m \gamma_{r,i,aa}^{edm} PMT_{r,i} XM_{r,i,aa} & \text{if } ArmFlag \neq 0 \end{cases} \quad (\text{Y-10})$$

<sup>15</sup> Goods at this level are assumed to be additive though allowances are made for different prices captured by the  $\gamma^e$  coefficients.

<sup>16</sup> The model allows for iceberg trade costs using the parameter  $\lambda^w$ . This generates a wedge between the volume exported and the volume imported. The import volume is given by  $XW^d$  and the export volume by  $XW^s$ . At equilibrium, the following expression holds:  $XW^d = \lambda^w XW^s$  and this expression is used to substitute out  $XW^d$ . Thus in the model implementation, the import tax revenue expression also contains the iceberg parameter as the model only carries the variable  $XW$  without a superscript and it represents export supply.

$$YGOV_{r,mtx} = \sum_s \sum_i \tau_{s,i,r}^m PWM_{s,i,r} XW_{s,i,r}^d \quad (\text{Y-11})$$

$$YGOV_{r,ctx} = \sum_d \sum_i \tau_{r,i,d}^e PE_{r,i,d} XW_{r,i,d}^s \quad (\text{Y-12})$$

$$YGOV_{r,ctx} = \begin{cases} \sum_{em} \sum_i \sum_{aa} \chi_{em}^{Emi} \rho_{r,em,i,aa}^{Emi} \phi_{r,em,i,aa}^{Emi} \tau_{r,em,aa}^{Emi} XA_{r,i,aa} & \text{if } ArmFlag = 0 \\ \sum_{em} \sum_i \sum_{aa} \chi_{em}^{Emi} \rho_{r,em,i,aa}^{Emi,d} \phi_{r,em,i,aa}^{Emi} \tau_{r,em,aa}^{Emi} XD_{r,i,aa} \\ \quad + \chi_{em}^{Emi} \rho_{r,em,i,aa}^{Emi,m} \phi_{r,em,i,aa}^{Emi} \tau_{r,em,aa}^{Emi} XM_{r,i,aa} & \text{if } ArmFlag \neq 0 \end{cases} \quad (\text{Y-13})$$

$$\begin{aligned} YGOV_{r,dtx} &= \sum_l \kappa_{r,l}^l \sum_a W_{r,l,a} L_{r,l,a}^d \\ &+ \kappa_r^k \left( \sum_a \left[ \sum_v PK_{r,a,v} K_{r,a,v}^v + \pi_{r,a}^m XP_{r,a} \right] - DeprY_r \right) \\ &+ \kappa_r^t \sum_a PLand_{r,a} Land_{r,a}^d \\ &+ \kappa_r^n \sum_a PNR_{r,a} XNR_{r,a}^d \\ &+ \kappa_r^w \sum_a PH2O_{r,a} H2O_{r,a}^d \\ &+ \kappa_r^h YH_r \end{aligned} \quad (\text{Y-14})$$

Equation (Y-15) describes the financing of gross investment. The variable  $YFD$  represents final demand expenditures, in value terms, for the final demand agents, indexed by  $fd$  that takes on values of  $h$ ,  $gov$  and  $inv$  respectively for households, government and investment. Gross investment is equated to the sum of all savings, domestic from households ( $S^h$ ) and government ( $S^g$ ), and foreign ( $S^f$ ), where the latter is evaluated using a global price index,  $PW^{sav}$ , and the depreciation allowance. Macro closure defines what variable this equation determines. In the default closure, investment is savings driven and therefore this equation determines the nominal level of investment. If investment is fixed, then this equation could determine either household or public savings. Macro closure is further discussed below. It is important to note that this equation is defined over all regions except for one that is the designated residual region. This is due to Walras' Law.<sup>17</sup>

$$YFD_{r,inv} = S_r^h + S_r^g + PW^{sav} S_r^f + DeprY_r \quad (\text{Y-15})$$

## 3.5 Final demand

### 3.5.1 Household demand

The commodity space for household demand has a different classification than the product commodity space (indexed by  $i$ ). Household demand for commodities is indexed by  $k$ . A transition matrix approach is used to convert the household commodities into the produced commodities.<sup>18</sup>

<sup>17</sup> In the model implementation, the residual region is a subset of  $r$  and is designated as **rres**.

<sup>18</sup> The consumer demand transition matrix approach was utilized in the OECD GREEN model, see Burniaux, Nicoletti, and Oliveira-Martins (1992) and van der Mensbrugge (1994). See also Cardenete, Guerra, and Sancho (2012), section 5.3, for a more recent description of the approach in GE modeling.



In value terms, the transition matrix is an  $i \times k$  matrix. The sum across the columns equals the value of consumption for commodity  $i$  and must line up with the values from the consumer demand vector of the SAM. The sum down the columns equals the value of consumption of commodity  $k$ . Table 3.3 provides an example from the GREEN database.<sup>19</sup>

Table 3.3: Example of a consumer transition matrix

	Food&Bev	Energy	Trp&Comm	Other	Total
Agric	15.9	0.1	0.0	5.2	21.3
Coal	0.0	0.3	0.0	0.0	0.3
Oil	0.0	0.0	0.0	0.0	0.0
Gas	0.0	12.8	0.0	0.0	12.8
Energy int manu	0.0	0.0	0.1	50.8	50.9
Refined oil	0.0	17.5	50.0	0.0	67.5
Electricity	0.0	73.1	0.0	22.9	96.0
Other	347.9	0.1	366.2	1,644.0	2,358.3
Total	363.9	103.8	416.3	1,723.0	2,607.0

There are several advantages to the transition matrix approach.

1. Potentially, it provides a more accurate picture of consumer demand. For example, it is possible to connect different energy bundles with different sources of demand. The demand for transport is likely to be dominated by liquid fuels, whereas the demand for energy in households is likely to be a mix of electricity, gas and coal.
2. It is likely to provide a better link with household surveys.
3. It may be easier to generate more plausible demand behavior. For example, in the standard LES demand system, a rise in the price of energy (due to a carbon tax for example) will normally lead to a rise in demand for automobiles. If fuel and cars are bundled in a transport bundle, the carbon tax will lead to a drop in the demand for the transport bundle and thus also a drop in the demand for cars.

With the original GTAP database, the transition matrix approach is largely limited to mapping specific supplied commodities to consumer commodities. At a minimum there would be a one-to-one mapping of all non-energy commodities, i.e. a diagonal transition matrix for the non-energy commodities, and all energy commodities mapped to a single energy consumer good.<sup>20</sup> However, with appropriate mapping it is possible to limit the number of consumer goods. For example, the user can map the GTAP commodities `tex`, `wap` and `lea` to a single 'clothing' consumer good and yet retain the production of all three commodities individually. In a similar vein, certain agriculture and food products can be aggregated together for the purposes of the top level consumer utility function, yet continue to be produced individually. The consumer demand nesting is then used to decompose the aggregate demand, for example for food or clothing, into their components.

The consumer demand system is thus implemented as a nested structure that starts with disposable income, see figure 7. A top nest allocates disposable income between savings and consumer

<sup>19</sup> The data is from the GREEN model database and represents the transition matrix for the USA in \$1985 million at the Armington level of aggregation.

<sup>20</sup> The user provides the relevant mappings between the GTAP goods (after aggregation) and the user-defined consumer goods. This mapping is part of the bridge file used by the aggregation facility.

commodities (indexed by  $k$ ). Each commodity  $k$  is decomposed into demand for the various product commodities (indexed by  $i$ ) using a series of nested CES functions. The top CES nest decomposes demand for commodity  $k$  into a non-energy aggregate bundle and an energy bundle. The former is decomposed into demand for non-energy Armington goods with a single nest (and added across all consumer goods  $k$ ). The latter is decomposed using the same energy nesting as in production with energy demand similarly aggregated across all consumer goods  $k$  to generate total energy demand by carrier by households.

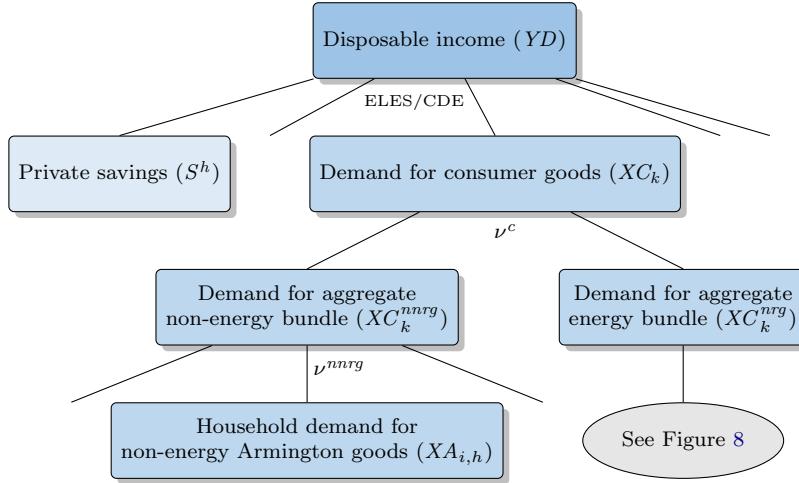


Figure 7: **Consumer demand nest**

The latest version of the ENVISAGE Model has four possible top-level consumer demand utility functions. Each can determine household demand for goods and services (at the level of consumer goods, i.e. indexed by  $k$ ). Three are in a class of models linked to the oft-used linear expenditure system (LES)—one of which, the ELES, integrates the savings decision in the utility function. The fourth utility function is the *constant differences in elasticity* (CDE) utility function—widely popularized in the GTAP model and its variants.<sup>21</sup>

### The Linear Expenditure System and its offshoots

This section describes use of the linear expenditure system (LES) and two of its offshoots. The first offshoot is the extended linear expenditure system (ELES) developed by Lluch (1973) and Howe (1975). The ELES integrates the savings decision into top level utility, i.e. the savings decision is handled with demand for the other goods and services. The second offshoot is the so-called "An Implicitly Direct Additive Demand System", or AIDADS, that leaves the possibility of non-linear Engel curves, unlike the LES—see Rimmer and Powell (1992a) and Rimmer and Powell (1996). All three demand systems share the same underlying specification with some deviations.

Under the LES-type demand system, household demand is the sum of two components—the so-called subsistence minimum, or floor consumption, and a marginal share of supernumerary income. The latter is equal to income less expenditures on the subsistence minima. Equation (D-1) defines supernumerary income on a per capita basis,  $Y^{sup}$ .<sup>22</sup> The first term is per capita income. In

<sup>21</sup> See Hertel (1997).

<sup>22</sup> The consumer demand system is written to allow for multiple households. However, at the moment, disposable income and population are only provided at an aggregate level. To convert the model to multiple households would require the development of a more complete income distribution specification.

the case of the the ELES, it is disposable income. In the case of the LES and AIDADS, it is disposable income less savings. The second term is the total amount spent on the subsistence minima where  $\theta$  represents the per capita subsistence minimum and expenditures are evaluated at consumer prices, all taxes included,  $PC$ . Equation (D-2) determines consumer demand for good  $k$ , where  $\mu^c$  represents the marginal budget share. Utility and parameters of the LES are defined in per capita terms. Hence, per capita demand is multiplied by population to get aggregate household demand. Equation (D-3) defines the utility level for the LES-type demand specification. In the case of the ELES, the variable  $P^{sav}$  represents the perceived 'price' of savings. Given the overall myopic expectations, it is equated to the consumer price index as it represents future claims on consumption. The parameter  $\mu^s$  represents the marginal propensity to save out of disposable income.

$$Y_{r,h}^{sup} = \begin{cases} \frac{YD_r}{Pop_r} - \sum PC_{r,k,h} \theta_{r,k,h} & \text{if ELES} \\ \frac{YD_r - S_{r,h}^k}{Pop_r} - \sum_k PC_{r,k,h} \theta_{r,k,h} & \text{if LES or AIDADS} \end{cases} \quad (\text{D-1})$$

$$XC_{r,k,h} = Pop_r \left( \theta_{r,k,h} + \frac{\beta_{r,h}^c \mu_{r,k,h}^c}{PC_{r,k,h}} Y_{r,h}^{sup} \right) \quad (\text{D-2})$$

$$u_{r,h} = \begin{cases} \frac{Y_{r,h}^{sup}}{\left( \frac{P^{sav}}{\mu_{r,h}^s} \right)^{\mu_{r,h}^s} \prod_k \left( \frac{PC_{r,k,h}}{\beta_{r,h}^c \mu_{r,k,h}^c} \right)^{\beta_{r,h}^c \mu_{r,k,h}^c}} & \text{if ELES} \\ -1 - \ln \left( A_{r,h}^{ad} \right) + \sum_k \mu_{r,k,h}^c \ln \left[ \frac{XC_{r,k,h}}{Pop_r} - \theta_{r,k,h} \right] & \text{if LES or AIDADS} \end{cases} \quad (\text{D-3})$$

The key difference between AIDADS and the standard LES is that the marginal budget share parameter,  $\mu^c$ , is a function of utility in the case of AIDADS, but is constant in the case of the LES. This allows for non-linear Engel curves as income expands. Equation (D-4) defines the marginal budget share specification. With  $\alpha^{ad}$  equal to  $\beta^{ad}$ , the function reverts back to the LES with constant marginal budget shares.

$$\mu_{r,k,h}^c = \frac{\alpha_{r,k,h}^{ad} + \beta_{r,k,h}^{ad} e^{u_{r,h}}}{1 + e^{u_{r,h}}} \quad (\text{D-4})$$

### The Constant Differences in Elasticity demand system

The CDE demand system is composed of four equations. The first equation, (D-5), evaluates an auxiliary variable,  $\theta$ , that is used in subsequent formulas and allows for some simplification in implementation of the CDE.<sup>23</sup> The  $e^h$  parameters are referred to as the expansion parameters and are closely linked to the income elasticities (see Appendix B). The  $b^h$  parameters are referred to as the substitution parameters and are linked to the price elasticities. The variable  $u$  is the utility level and is required for the CDE demand system. The  $\alpha^h$  parameters are calibrated shifters. Equation (D-6) determines the budget shares for the CDE. Equation (D-7) implicitly determines utility that is required to calculate the budget shares. And equation (D-8) is used to back out

<sup>23</sup>  $\theta$  is used as a variable in the CDE demand system. In the ELES,  $\theta$  represents subsistence consumption and is normally treated as a parameter or it is exogenous.

consumption levels using the budget shares. Note that equation (D-8) is also used in the LES-type demand systems to calculate the budget shares—though these are not strictly needed.<sup>24</sup>

$$\theta_{r,k,h} = \alpha_{r,k,h}^h b_{r,k,h}^h u_{r,h}^{e_{r,k,h}^h b_{r,k,h}^h} \left( \frac{PC_{r,k,h}}{(YD_r - S_r^h)/Pop_r} \right)^{b_{r,k,h}^h} \quad (D-5)$$

$$s_{r,k,h}^h = \frac{\theta_{r,k,h}}{\sum_{k'} \theta_{r,k',h}} \quad (D-6)$$

$$\sum_k \frac{\theta_{r,k,h}}{b_{r,k,h}^h} \equiv 1 \quad (D-7)$$

$$s_{r,k,h}^h = \frac{PC_{r,k,h} XC_{r,k,h}}{YD_r - S_r^h} \quad (D-8)$$

### Conversion of consumer goods to producer goods

The subsequent nests decompose consumer demand  $XC$  into Armington demand and the latter are summed across all consumer categories  $k$ . The first nest allocates  $XC$  between a non-energy bundle and an energy bundle. Equations (D-9) and (D-10) describe respectively the demand for the aggregate non-energy and energy bundles,  $XC^{nnrg}$  and  $XC^{nrg}$ , with a substitution elasticity given by  $\nu^c$ . Equation (D-11) represents the price of the consumer good defined by index  $k$ ,  $PC$ .

$$XC_{r,k,h}^{nnrg} = \alpha_{r,k,h}^{cnrg} \left( \frac{PC_{r,k,h}}{PC_{r,k,h}^{nnrg}} \right)^{\nu_{r,k,h}^c} XC_{r,k,h} \quad (D-9)$$

$$XC_{r,k,h}^{nrg} = \alpha_{r,k,h}^{cnrg} \left( \frac{PC_{r,k,h}}{PC_{r,k,h}^{nrg}} \right)^{\nu_{r,k,h}^c} XC_{r,k,h} \quad (D-10)$$

$$PC_{r,k,h} = \left[ \alpha_{r,k,h}^{cnrg} (PC_{r,k,h}^{nnrg})^{1-\nu_{r,k,h}^c} + \alpha_{r,k,h}^{cnrg} (PC_{r,k,h}^{nrg})^{1-\nu_{r,k,h}^c} \right]^{1/(1-\nu_{r,k,h}^c)} \quad (D-11)$$

Aggregate non-energy demand is decomposed next into Armington demand and summed across all consumer goods. Equation (D-12) represents demand for non-energy Armington goods by private households, where the across-goods substitution is given by  $\nu^{nnrg}$ .<sup>25</sup> The price of the non-energy demand bundle is given by equation (D-13).

$$XA_{r,in,h} = \sum_k \left[ \alpha_{r,in,k,h}^c \left( \frac{PC_{r,k,h}^{nnrg}}{PA_{r,in,h}^a} \right)^{\nu_{r,k,h}^{nnrg}} XC_{r,k,h}^{nnrg} \right] \quad (D-12)$$

$$PC_{r,k,h}^{nnrg} = \left[ \sum_{in} \alpha_{r,in,k,h}^c (PA_{r,in,h}^a)^{1-\nu_{r,k,h}^{nnrg}} \right]^{1/(1-\nu_{r,k,h}^{nnrg})} \quad (D-13)$$

The decomposition of the energy bundle is similar to the energy nest in production, see figure 8. The energy bundle is first decomposed into electric and non-electric bundles. The latter is then

<sup>24</sup> Note that the budget shares reflect the shares net of savings. In the case of the ELES, the shares needed for calibration are relative to disposable income, i.e. inclusive of savings.

<sup>25</sup> The index  $in$  covers the non-energy commodities and is a subset of  $i$ .

decomposed into a coal bundle and a non-coal bundle (or the oil & gas bundle). The oil & gas bundle is then split into a gas bundle and an oil bundle. The four remaining bundles—electric, coal, oil and gas—represent a combination of existing or future energy sources. In the case of the original GTAP database, the electric bundle would hold the 'ely' commodity, the coal bundle would hold the 'coa' commodity, the oil bundle would hold the 'oil' and 'p\_c' commodities and the gas bundle would hold the 'gas' and 'gdt' commodities. Non-GTAP commodities would be mapped to one of the existing bundles. For example, 'p\_c' could be split into gasoline and diesel, and/or could include ethanol or bio-diesel.

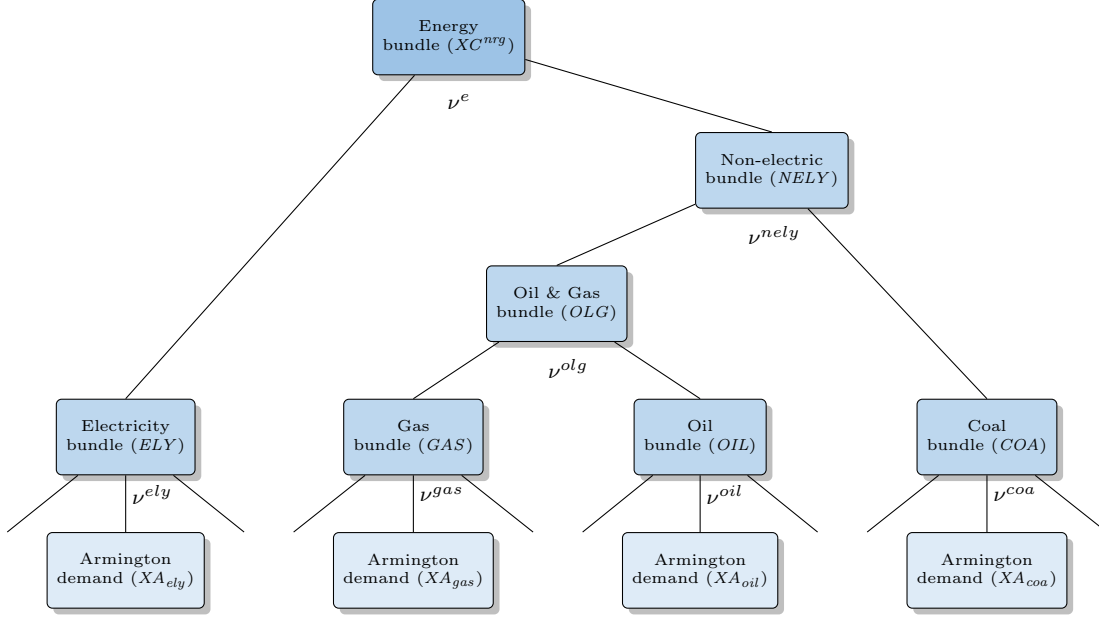


Figure 8: Energy bundle nest in household demand

Equation (D-14) determines the demand for the electric bundle,  $XC^{ely}$ .<sup>26</sup> Equation (D-15) determines the demand for the non-electric bundle,  $XC^{nely}$ . In both equations, the key substitution elasticity is given by  $\nu^e$ . Equation (D-16) then describes the aggregate price of energy,  $PC^{nrq}$ .

$$XC_{r,k,h}^{ely} = \alpha_{r,k,h}^{cely} \left( \frac{PC_{r,k,h}^{nrq}}{PC_{r,k,h}^{ely}} \right)^{\nu_{r,k,h}^e} XC_{r,k,h}^{nrq} \quad (\text{D-14})$$

$$XC_{r,k,h}^{nely} = \alpha_{r,k,h}^{cnely} \left( \frac{PC_{r,k,h}^{nrq}}{PC_{r,k,h}^{nely}} \right)^{\nu_{r,k,h}^e} XC_{r,k,h}^{nrq} \quad (\text{D-15})$$

$$PC_{r,k,h}^{nrq} = \left[ \alpha_{r,k,h}^{cely} \left( PC_{r,k,h}^{ely} \right)^{1-\nu_{r,k,h}^e} + \alpha_{r,k,h}^{cnely} \left( PC_{r,k,h}^{nely} \right)^{1-\nu_{r,k,h}^e} \right]^{\frac{1}{1-\nu_{r,k,h}^e}} \quad (\text{D-16})$$

Equation (D-17) determines the demand for the coal bundle,  $XC^{coa}$ . Equation (D-18) determines the demand for the oil & gas bundle,  $XC^{olg}$ . In both equations, the key substitution elasticity

<sup>26</sup> In the GAMS code, the demand for the four energy bundles ('ely', 'gas', 'oil' and 'coa') are represented by the variable `xacNRG` that has an additional dimension representing the four main energy carriers. This simplifies the decomposition of these bundles that only requires one set of equations rather than four. In principle, it also makes it somewhat easier to increase the number of energy bundles.

is given by  $\nu^{nely}$ . Equation (D-19) then describes the aggregate price of the non-electric bundle,  $PC^{nely}$ .

$$XC_{r,k,h}^{coa} = \alpha_{r,k,h}^{coa} \left( \frac{PC_{r,k,h}^{nely}}{PC_{r,k,h}^{coa}} \right)^{\nu_{r,k,h}^{nely}} XC_{r,k,h}^{nely} \quad (D-17)$$

$$XC_{r,k,h}^{olg} = \alpha_{r,k,h}^{colg} \left( \frac{PC_{r,k,h}^{nely}}{PC_{r,k,h}^{olg}} \right)^{\nu_{r,k,h}^{nely}} XC_{r,k,h}^{nely} \quad (D-18)$$

$$PC_{r,k,h}^{nely} = \left[ \alpha_{r,k,h}^{coa} (PC_{r,k,h}^{coa})^{1-\nu_{r,k,h}^{nely}} + \alpha_{r,k,h}^{colg} (PC_{r,k,h}^{olg})^{1-\nu_{r,k,h}^{nely}} \right]^{\frac{1}{1-\nu_{r,k,h}^{nely}}} \quad (D-19)$$

The remaining two energy bundles are oil and gas and emanate from the  $XC^{olg}$  bundle. Equation (D-20) determines the demand for the oil bundle,  $XC^{oil}$ . Equation (D-21) determines the demand for the gas bundle,  $XC^{gas}$ . In both equations, the key substitution elasticity is given by  $\nu^{olg}$ . Equation (D-22) then describes the aggregate price of the oil & gas bundle,  $PC^{olg}$ .

$$XC_{r,k,h}^{oil} = \alpha_{r,k,h}^{coil} \left( \frac{PC_{r,k,h}^{olg}}{PC_{r,k,h}^{oil}} \right)^{\nu_{r,k,h}^{olg}} XC_{r,k,h}^{olg} \quad (D-20)$$

$$XC_{r,k,h}^{gas} = \alpha_{r,k,h}^{cgas} \left( \frac{PC_{r,k,h}^{olg}}{PC_{r,k,h}^{gas}} \right)^{\nu_{r,k,h}^{olg}} XC_{r,k,h}^{olg} \quad (D-21)$$

$$PC_{r,k,h}^{olg} = \left[ \alpha_{r,k,h}^{coil} (PC_{r,k,h}^{oil})^{1-\nu_{r,k,h}^{olg}} + \alpha_{r,k,h}^{cgas} (PC_{r,k,h}^{gas})^{1-\nu_{r,k,h}^{olg}} \right]^{\frac{1}{1-\nu_{r,k,h}^{olg}}} \quad (D-22)$$

The final nest in the energy bundle is to decompose the four aggregate energy bundles into their constituent parts that represent the Armington demand for the energy commodities for each commodity  $k$  and are then summed across all consumer commodities. Equation (D-23) reflects the Armington demand for energy commodity  $e$ ,  $XA_h$ , where the cost to consumers is given by  $PA_h^a$ . The key substitution elasticity for each energy bundle is given by  $\nu^{NRG}$ . Equation (D-24) represents the price of the aggregate energy bundles,  $PC^{NRG}$ .

$$XA_{r,e,h} = \sum_k \alpha_{r,e,k,h}^c \left( \frac{\lambda_{r,e,k,h}^{ce} PC_{r,k,h}^{NRG}}{PA_{r,e,h}^a} \right)^{\nu_{r,k,h}^{NRG}} \frac{XC_{r,k,h}^{NRG}}{\lambda_{r,e,k,h}^{ce}} \quad \text{if } e \in \{NRG\} \quad (D-23)$$

$$PC_{r,k,h}^{NRG} = \left[ \sum_{e \in \{NRG\}} \alpha_{r,e,k,h}^c \left( \frac{PA_{r,e,h}^a}{\lambda_{r,e,k,h}^{ce}} \right)^{1-\nu_{r,k,h}^{NRG}} \right]^{\frac{1}{1-\nu_{r,k,h}^{NRG}}} \quad (D-24)$$

### Household savings

Household savings is treated differently across the demand systems. In the case of the ELES, savings is incorporated in the utility function. In the case of the other demand systems, a separate assumption is made about the savings propensity and savings is subtracted from disposable income with the residual allocated to consumer goods.

Equation (D-25) determines savings for all utility function with the exception of the ELES. Savings is simply a proportion of household disposable income. The initial savings rate is given by  $aps$  and the savings rate shifter variable,  $\chi^s$  is equal, to set to 1 initially. Depending on closure, the variable  $\chi^s$  can be endogenous to target some other objective, for example, the level of investment relative to GDP. In other closures, it could be exogenous, in which case investment is savings driven. In the case of the ELES, the equation in essence determines  $\chi^s$ , i.e. the average shifter of the savings rate, since savings is determined simultaneously with consumption. In the case of the ELES, equation (D-26) determines household savings as the difference between disposable income and total expenditures on goods and services.<sup>27</sup> In the case of the other demand systems, the equation is dropped.

$$S_{r,h}^h = \chi_r^s aps_r YD_r \quad (\text{D-25})$$

$$S_{r,h}^h = YD_r - \sum_k PC_{r,k,h} XC_{r,k,h} \quad (\text{D-26})$$

### 3.5.2 Other final demand

In the standard model there are two other domestic agents—government and investment. The demand for goods and services for each is governed by a CES expenditure function. Equation (D-27) represents the demand, therefore, for the remaining final demand agents, indexed by  $fdc$ . The substitution elasticity across goods is given by  $\sigma^{fd}$  and will typically be either 0 or 1, i.e. either constant volume shares or constant value shares. Equation (D-28) determines the other final demand aggregate price deflator,  $PFD_{fdc}$ , using the ubiquitous CES dual price expression. Equation (D-29) is an identity relating nominal expenditures to real expenditures. What equation (D-29) identifies will depend on macro closure. In the standard model, it provides a definition of the volume of consumer expenditures (where the price index is defined elsewhere), the nominal level of government expenditures where by default the volume of government expenditures is exogenous, and the volume of investment expenditures, where the nominal level is determined by the investment-savings balance equation, equation (Y-15) above.

$$XA_{r,i,fdc} = \alpha_{r,i,fdc}^{fd} \left( \frac{PFD_{r,fdc}}{PA_{r,i,fdc}^a} \right)^{\sigma_{r,fdc}^{fd}} XFD_{r,fdc} \quad (\text{D-27})$$

$$PFD_{r,fdc} = \left[ \sum_i \alpha_{r,i,fdc}^{fd} (PA_{r,i,fdc}^a)^{1-\sigma_{r,fdc}^{fd}} \right]^{1/(1-\sigma_{r,fdc}^{fd})} \quad (\text{D-28})$$

$$YFD_{r,fd} = PFD_{r,fd} XFD_{r,fd} \quad (\text{D-29})$$

## 3.6 International trade

### 3.6.1 Import demand

Similar to most CGE models, import demand uses the Armington specification that assumes product differentiation between domestic and imported goods. A nested CES structure first differenti-

<sup>27</sup> To target investment, in the case of the ELES it is possible to target a given level of investment (as a share of GDP for example) and to endogenize the  $\beta^c$  parameter that in effect will find a level of savings consistent with targeted investment.

ates between domestic goods and aggregate imports. A second nest differentiates imports across source regions. The GTAP database and the standard GTAP model assume that the top level Armington nest is agent-specific, i.e. the allocation of Armington demand between domestic goods and aggregate imports. The ENVISAGE model allows for 'national' sourcing, i.e. that the top level Armington disaggregation is done after aggregating Armington demand across all agents. While the former is perhaps a more plausible assumption under certain circumstances, this formulation requires significantly more computation. The top level sourcing is governed by *ArmFlag*. If it is set to zero, national sourcing is assumed. Any other value implies agent-specific sourcing.<sup>28</sup>

### National sourcing of aggregate imports

This first section describes the top level Armington equations in the case of national sourcing. Equation (T-1) adds up Armington demand (or domestic absorption for the Armington good) across all Armington agents to measure aggregate Armington demand,  $XAT$ . The latter is then decomposed into demand for domestically produced goods,  $XDT^d$ , equation (T-2), and aggregate import demand,  $XMT$ , equation (T-3). The parameter  $\sigma^{mt}$  represents the key substitution elasticity and is often referred to as the Armington elasticity. Each region in the model provides international trade and transport services  $XTT$ , further described below. It is assumed that these services are only provided using domestically produced goods. Hence, equation (T-2) also includes the region's provision of these services, in addition to the demand for domestic goods coming from the other agents in the economy. Equation (T-4) determines the Armington price (of the aggregate Armington bundle),  $PAT$ , a CES combination of the domestic price and the aggregate import price.

$$XAT_{r,i} = \sum_{aa} \gamma_{r,i,aa}^{eda} XA_{r,i,aa} \quad (\text{T-1})$$

$$XDT_{r,i}^d = \alpha_{r,i}^{dt} \left( \frac{PAT_{r,i}}{PDT_{r,i}} \right)^{\sigma_{r,i}^{mt}} XAT_{r,i} + XTT_{r,i} \quad (\text{T-2})$$

$$XMT_{r,i} = \alpha_{r,i}^{mt} \left( \frac{PAT_{r,i}}{PMT_{r,i}} \right)^{\sigma_{r,i}^{mt}} XAT_{r,i} \quad (\text{T-3})$$

$$PAT_{r,i} = \left[ \alpha_{r,i}^{dt} (PDT_{r,i})^{1-\sigma_{r,i}^{mt}} + \alpha_{r,i}^{mt} (PMT_{r,i})^{1-\sigma_{r,i}^{mt}} \right]^{1/(1-\sigma_{r,i}^{mt})} \quad (\text{T-4})$$

Note that the zero profit condition is given by:

$$PAT_{r,i} XAT_{r,i} = PDT_{r,i} \left( XDT_{r,i}^d - XTT_{r,i} \right) + PMT_{r,i} XMT_{r,i}$$

Agents pay  $PA$  for Armington goods, as defined by equation (T-5), where  $\tau^a$  is the agent specific tax on Armington consumption. The economy-wide Armington price,  $PAT$ , is allowed to vary across end-users using a price wedge represented by  $\gamma^{eda}$ . The emissions tax is treated as a Pigouvian tax that is added to the after-tax Armington price. The parameter  $\rho^{Emi}$  represents the rate of emissions per unit of consumption and  $\tau^{Emi}$  is the emissions tax per unit of emissions. The parameter  $\chi^{Emi}$  is a global shifter on the emission rates. And the parameter  $\phi^{Emi}$  allows for partial or full exemption of the tax incidence. By default,  $\phi^{Emi}$  is set to 1. Any valid value is between 0

<sup>28</sup> The allocation of aggregate imports across source regions is done at the national level. Full sourcing by region requires a so-called multi-regional input-output (MRIO) table and is also computational intensive.



(full exemption) to 1 (zero exemption). For example specific activities and/or households can be exempted from the emissions tax and for specific fuels as well—for example natural gas.

$$PA_{r,i,aa} = (1 + \tau_{r,i,aa}^a) \gamma_{r,i,aa}^{eda} PA_{r,i} + \sum_{em} \chi_{em}^{Emi} \rho_{r,em,i,aa}^{Emi} \phi_{r,em,i,aa}^{Emi} \tau_{r,em,aa}^{Emi} \quad (\text{T-5})$$

This section determines  $XAT$ ,  $XDT$ ,  $XMT$ ,  $PAT$ , and  $PA$ . The variables  $XAT$  and  $PAT$  are specific to the national-sourcing assumption and are not part of the model specification in the case of agent-sourcing. The other variables are included in the agent-sourcing specification, but have an alternative specification.

### Agent sourcing of aggregate imports

In the agent-sourcing of imports, each agent has a unique preference for domestic goods and (aggregate) imports. This specification is more computationally intensive, but is likely to be a more plausible specification. Equations (T-6) and (T-7) determine respectively the agents' demand for domestic goods and the aggregate import bundle, respectively  $XD$  and  $XM$ . All relevant variables and parameters are indexed by  $aa$ , which represents the set of all Armington agents. Equation (T-8) defines the agent-specific Armington price, which corresponds to  $PA$  above, but where the agent-specificity was due simply to the end-user sales tax since all agents were faced with the same top-level Armington price.

$$XD_{r,i,aa} = \alpha_{r,i,aa}^d \left( \frac{PA_{r,i,aa}}{PD_{r,i,aa}} \right)^{\sigma_{r,i,aa}^m} XA_{r,i,aa} \quad (\text{T-6})$$

$$XM_{r,i,aa} = \alpha_{r,i,aa}^m \left( \frac{PA_{r,i,aa}}{PM_{r,i,aa}} \right)^{\sigma_{r,i,aa}^m} XA_{r,i,aa} \quad (\text{T-7})$$

$$PA_{r,i,aa} = \left[ \alpha_{r,i,aa}^d (PD_{r,i,aa})^{1-\sigma_{r,i,aa}^m} + \alpha_{r,i,aa}^m (PM_{r,i,aa})^{1-\sigma_{r,i,aa}^m} \right]^{1/(1-\sigma_{r,i,aa}^m)} \quad (\text{T-8})$$

The next two equations define the end-user, or agent-specific, sales price, equations (T-9) and (T-10). Similar to the specification of the Armington price with national sourcing, all agents face the same (aggregate) market price for goods, respectively  $PDT$  and  $PMT$ . The prices can be differentiated by agent with exogenous wedges ( $\gamma^e$ ) and agent-specific sales taxes. The carbon tax is a Pigouvian tax added to the tax-inclusive sales price, where the emission coefficient is source-specific.

$$PD_{r,i,aa} = (1 + \tau_{r,i,aa}^d) \gamma_{r,i,aa}^{edd} PDT_{r,i} + \sum_{em} \chi_{em}^{Emi} \rho_{r,em,i,aa}^{Emi,d} \phi_{r,em,i,aa}^{Emi} \tau_{r,em,aa}^{Emi} \quad (\text{T-9})$$

$$PM_{r,i,aa} = (1 + \tau_{r,i,aa}^m) \gamma_{r,i,aa}^{edm} PMT_{r,i} + \sum_{em} \chi_{em}^{Emi} \rho_{r,em,i,aa}^{Emi,m} \phi_{r,em,i,aa}^{Emi} \tau_{r,em,aa}^{Emi} \quad (\text{T-10})$$

The final two equations determine aggregate demand for domestic goods,  $XDT$ , and the import bundle,  $XMT$ . These correspond to equation (T-1) above and represent a simple aggregation of the agent-specific demand (adjusted by the agent-specific coefficients to allow for exogenous price relatives). Note that domestic supply of international trade and transport margins is added to the (Armington) agents' demand for domestic goods.

$$XDT_{r,i}^d = \sum_{aa} \gamma_{r,i,aa}^{edd} XD_{r,i,aa} + XTT_{r,i} \quad (\text{T-11})$$

$$XMT_{r,i} = \sum_{aa} \gamma_{r,i,aa}^{edm} XM_{r,i,aa} + XTT_{r,i} \quad (\text{T-12})$$

This section determines  $XD$ ,  $XM$ ,  $PA$ ,  $PD$ ,  $PM$ ,  $XDT^d$  and  $XMT$ . The variables  $PA$ ,  $XDT^d$  and  $XMT$  are common to both specifications albeit with different expressions. The variables  $XD$ ,  $XM$ ,  $PD$  and  $PM$  are only included with the agent-sourcing specification.

### Second level Armington nest

The second CES nest decomposes the aggregate demand for imports,  $XMT$ , into demand for imports by source region,  $XW^d$ . Equation (T-13) describes demand for imports by region  $r$  for imports from region  $s$  for good  $i$ .<sup>29</sup> The variable  $PDM$  represents the end-user price of imports, i.e. it includes bilateral tariffs. The key substitution elasticity is given by  $\sigma^w$ . The aggregate import price,  $PMT$ , is the CES aggregation of the tariff-inclusive bilateral prices as described in equation (T-14).

$$XW_{s,i,r}^d = \alpha_{s,i,r}^w \left( \frac{PMT_{r,i}}{PDM_{s,i,r}} \right)^{\sigma_{r,i}^w} XMT_{r,i} \quad (\text{T-13})$$

$$PMT_{r,i} = \left[ \sum_s \alpha_{s,i,r}^w (PDM_{s,i,r})^{1-\sigma_{r,i}^w} \right]^{1/(1-\sigma_{r,i}^w)} \quad (\text{T-14})$$

### 3.6.2 Export supply

Domestic supply of good  $i$  is given by  $XS$ . It will be allocated between domestic and export markets analogously to the Armington assumption on the demand side. At the top level, the national supplier decides the optimal allocation of total supply between the domestic market and aggregate exports. At the second level, aggregate exports are allocated across all regions of destination. The allocation functions are based on the CET transformation specification. The implementation does allow for perfect transformation, in which case the law-of-one-price holds.

Equation (T-15) determines the optimal supply of domestic production to be allocated to the domestic market,  $XDT^s$ . The transformation elasticity is given by  $\omega^x$ . In the case of perfect transformation, the market price of goods sold domestically is equal to the average supply price adjusted by an exogenous price wedge,  $\gamma^{esd}$ . Equation (T-16) determines the optimal supply of aggregate exports,  $XET$ . Equation (T-17) represents the zero-profit condition and is in some sense the equilibrium condition for total supply,  $XS$ .<sup>30</sup>

$$\begin{cases} XDT_{r,i}^s = \gamma_{r,i}^d (\gamma_{r,i}^{esd})^{-\omega_{r,i}^x - 1} \left( \frac{PDT_{r,i}}{PS_{r,i}} \right)^{\omega_{r,i}^x} XS_{r,i} & \text{if } \omega_{r,i}^x \neq \infty \\ PDT_{r,i} = \gamma_{r,i}^{esd} PS_{r,i} & \text{if } \omega_{r,i}^x = \infty \end{cases} \quad (\text{T-15})$$

$$\begin{cases} XET_{r,i} = \gamma_{r,i}^e (\gamma_{r,i}^{ese})^{-\omega_{r,i}^x - 1} \left( \frac{PET_{r,i}}{PS_{r,i}} \right)^{\omega_{r,i}^x} XS_{r,i} & \text{if } \omega_{r,i}^x \neq \infty \\ PET_{r,i} = \gamma_{r,i}^{ese} PS_{r,i} & \text{if } \omega_{r,i}^x = \infty \end{cases} \quad (\text{T-16})$$

$$PS_{r,i} XS_{r,i} = PDT_{r,i} XDT_{r,i}^s + PET_{r,i} XET_{r,i} \quad (\text{T-17})$$

<sup>29</sup> As mentioned above, the  $XW^d$  is substituted out and thus this expression carries the iceberg parameter in the model implementation.

<sup>30</sup> It could be replaced with either the CET primal or dual expressions.

The second level nest is represented by equation (T-18), which determines the quantity of exports allocated to each of the destination markets ( $d$ ),  $XW^s$ . The key transformation elasticity is given by  $\omega^w$ . Equation (T-19) determines the aggregate export price,  $PET$ . Note that when the elasticity is infinite, the prices in this equation are all identical and thus the equation holds in both volume and value terms. Similar to the top-level CET nest, the second level nest allows for an exogenous wedge in the bilateral prices relative to the average, or aggregate, price.

$$\begin{cases} XW_{r,i,d}^s = \gamma_{r,i,d}^w (\gamma_{r,i,d}^{ew})^{-\omega_{r,i}^w - 1} \left( \frac{PE_{r,i,d}}{PET_{r,i}} \right)^{\omega_{r,i}^w} XET_{r,i} & \text{if } \omega_{r,i}^w \neq \infty \\ PE_{r,i,d} = \gamma_{r,i,d}^{ew} PET_{r,i} & \text{if } \omega_{r,i}^w = \infty \end{cases} \quad (\text{T-18})$$

$$PET_{r,i} XET_{r,i} = \sum_d PE_{r,i,d} XW_{r,i,d}^s \quad (\text{T-19})$$

### 3.6.3 Bilateral trade prices

Each bilateral trade node is associated with four prices.  $PE$  represents the price domestic producers in region  $r$  receive for goods exported to destination region  $d$ . An export tax or subsidy,  $\tau^e$ , is introduced between the producer price and the border (of FOB) price,  $PWE$ , equation (T-20). A transportation margin represents the wedge between the border (or FOB) price of the exporting region and the border (or CIF) price of the importing region,  $PWM$ . The variable  $\zeta^{mg}$  represents the per unit transportation margin that is valued at the average price of  $PWMG$ , equation (T-21). The CIF import price is adjusted by the iceberg parameter,  $\lambda^w$ . The final price of imports for the end-user is the border (or CIF) price plus the bilateral import tariff  $\tau^m$ . This price is represented by  $PDM$ , equation (T-22).<sup>31</sup>

$$PWE_{r,i,d} = (1 + \tau_{r,i,d}^e) PE_{r,i,d} \quad (\text{T-20})$$

$$PWM_{r,i,d} = \left[ PWE_{r,i,d} + PWMG_{r,i,d} \zeta_{r,i,d}^{mg} \right] / \lambda_{r,i,d}^w \quad (\text{T-21})$$

$$PDM_{r,i,d} = (1 + \tau_{r,i,d}^m) PWM_{r,i,d} \quad (\text{T-22})$$

### 3.6.4 Trade margins

International trade is associated with transport margins that capture the wedge between the price at the source port and the price at the destination port, i.e. the CIF/FOB price wedge. Thus trade engenders a demand for services. The supply of these services is assumed to emanate from a global supplier that chooses the lowest cost supply subject to a CES preference function.

Equation (T-23) provides the demand for international trade and transport services per bilateral node,  $XWMG$ , a simple linear technology. The bilateral demand for trade and transport services is allocated across margin commodities ( $m$ ) using a similar linear technology, equation (T-24). The specification allows for technological change as rendered by the variable  $\lambda^{mg}$ . The average price of transportation per bilateral node,  $PWMG$ , is given by equation (T-25).

$$XWMG_{r,i,d} = \zeta_{r,i,d}^{mg} XW_{r,i,d}^s \quad (\text{T-23})$$

<sup>31</sup> Note that in the model implementation, these equations are typically substituted out of the model in order to reduce its overall size.

$$XMG_{m,r,i,d}^m = \alpha_{m,r,i,d}^{mg} \frac{XWMG_{r,i,d}}{\lambda_{m,r,i,d}^{mg}} \quad (\text{T-24})$$

$$PWMG_{r,i,d} = \sum_m \alpha_{m,r,i,d}^{mg} \frac{PTMG_m}{\lambda_{m,r,i,d}^{mg}} \quad (\text{T-25})$$

Total global demand (and therefore supply) for trade and transport margins,  $XTMG$ , for each commodity  $m$ , is the sum across all potential bilateral nodes equation (T-26). The global supplier allocates this demand across potential suppliers using a CES preference function. Equation (T-27) determines region  $r$ 's supply of trade and transport services,  $XTT$  for commodity  $m$ . The global average price for trade and transport service  $m$ ,  $PTMG$ , is given by equation (T-28).

$$XTMG_m = \sum_r \sum_i \sum_d XMG_{m,r,i,d}^m \quad (\text{T-26})$$

$$XTT_{r,m} = \alpha_{r,m}^{tt} \left( \frac{PTMG_m}{PD_{r,m}} \right)^{\sigma_m^{mg}} XTMG_m \quad (\text{T-27})$$

$$PTMG_m XTMG_m = \sum_m PD_{r,m} XTT_{r,m} \quad (\text{T-28})$$

### 3.7 Goods market equilibrium

There are two sets of equilibrium conditions. The domestic market for goods produced domestically and bilateral trade. Equation (E-1) determines  $PD$ , the market or equilibrium price equating supply of domestically produced goods to the demand for domestic goods. Equation (E-2) determines  $PE$ , the producer price of exported goods by region of origin  $r$  destined for region  $d$ . The equilibrium condition allows for iceberg trade costs that are represented by the parameter  $\lambda^w$ . The parameter is initialized at 1. An improvement in trade, i.e. a reduction in iceberg costs, is represented by an increase from 1. Thus a 10 percent improvement is represented by setting  $\lambda^w$  to 1.1. In model implementation, these equilibrium conditions are substituted out of the model specification.

$$XDT_{r,i}^d = XDT_{r,i}^s \quad (\text{E-1})$$

$$XW_{r,i,d}^d = \lambda_{r,i,d}^w XW_{r,i,d}^s \quad (\text{E-2})$$

### 3.8 Factor markets

#### 3.8.1 Labor market

Equilibrium on the labor markets allows for labor market segmentation—rural, equated with agricultural sectors, and urban—or full mobility across all sectors. In the case of labor market segmentation, the model also implements a migration function that allows for some adjustment in the respective labor supplies depending on expected relative wage rates. The two labor market segments are indexed by  $z$ , or zone. When market segmentation is assumed, the index  $z$  has two elements, **rur** and **urb**. For nationally integrated labor markets, the index  $z$  has a single element, **nsg** (i.e. not-segmented). Note that the model allows for mixed specifications. For example, market segmentation can be assumed for unskilled workers but skilled workers could be fully mobile.

Within each zone (in either the segmented or non-segmented specifications), clearance of the labor market allows for less than full employment. A reservation (or minimum) wage is defined that puts a lower bound on market wages. With sufficiently high demand the equilibrium wage will likely exceed the reservation wage and the wage will be determined by equality of demand and supply (allowing for 'natural' unemployment). If labor demand is low, the market wage would be determined by the reservation wage and labor supply will exceed labor demand generating unemployment.

Equation (F-1) defines labor demand within the relevant labor market zone.<sup>32</sup>

$$LD_{r,l,z}^z = \sum_{a \in z} L_{r,l,a}^d \quad (\text{F-1})$$

Labor market closure allows for endogenous regime shifting between full employment with the equilibrium wage determined by the equality of supply and demand and an a regime of less than full employment. A reservation (or minimum) wage puts a floor on wages. If labor demand is low relative to supply, the market wage will be determined by the reservation wage. In the case of high labor demand, wages will be determined by equilibrium conditions. The two regimes are specified using a mixed complementarity expression that takes the following form:

$$(W^e - W^{res})(UE - UE^{min}) = 0$$

with

$$W^e \geq W^{res} \quad \text{and} \quad UE \geq UE^{min}$$

where  $W^e$  is the market wage,  $W^{res}$  is the reservation wage,  $UE$  is the rate of unemployment and  $UE^{min}$  is a floor for the unemployment rate. For the complementarity condition to hold either the unemployment rate is at the minimum and the market wage is greater than the reservation wage, or, the unemployment level is greater than the minimum level and the market wage is equal to the reservation wage.

Figure 9 provides an illustration of the two different market closures. The vertical line labeled  $L^s$  represents total labor supply, however effective labor supply is represented by the other vertical line,  $(1 - UE^{min})L^s$ , and represents labor supply adjusted by the minimal (or natural) rate of unemployment (that is exogenous). The labor demand curves  $D_1D_1$  and  $D_2D_2$  represent two different situations with the former having relatively low labor demand and the latter relatively high labor demand. The  $D_1D_1$  curve crosses the effective labor supply curve below the reservation wage and hence the market wage will be given by the reservation wage as indicated at point A. The rate of unemployment is measured as the distance between total labor supply and effective labor demand ( $L_1^d$ ), of which some part reflects the minimum rate of unemployment. The  $D_2D_2$  curve, on the other hand, crosses the effective labor supply curve at a wage greater than the reservation wage, in which case the market wage is given by supply/demand equality at point B and with a market wage of  $W^e$ . Effective labor demand is  $L_2^d$  and equal to effective labor supply. As the demand curve  $D_1D_1$  shifts up and to the right, the market wage will be determined by the reservation wage until point C when the regime shift towards full employment occurs.

Equation (F-2) defines the reservation wage,  $W^{res}$ . It is a positive function of income growth per capita and the CPI. It is a negative function of the unemployment rate, i.e. if the rate of unemployment increases, this would tend to have a downward pressure on the reservation wage.

<sup>32</sup> All equations relating to labor market zones are conditioned on a set flag represented by `lsFlag`. For segmented labor markets, the flag will be active for `rur` and `urb`. For non-segmented labor markets, the flag will only be active for `nsg`.

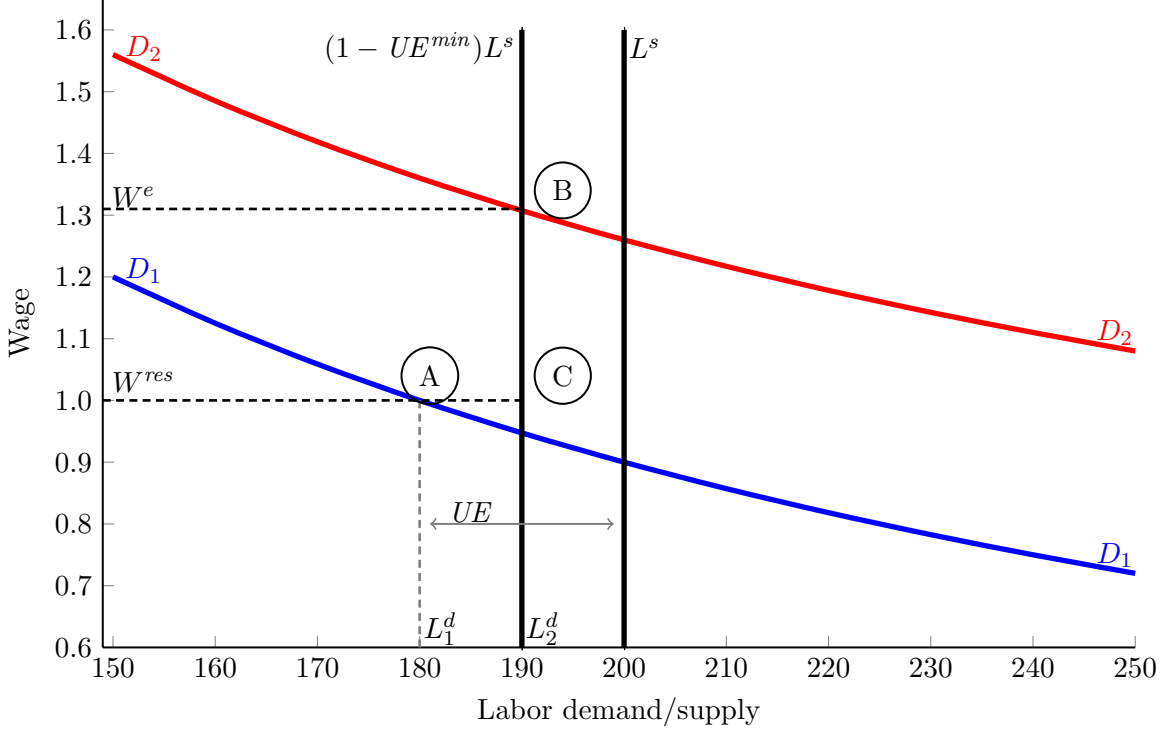


Figure 9: Labor market closure

The (positive) elasticities  $\omega^{rwg}$  and  $\omega^{rwp}$  reflect the sensitivity of the reservation wage with respect to income growth and the CPI, respectively. The (positive) elasticity  $\omega^{rwe}$  reflects the sensitivity of the reservation wage with respect to the rate of unemployment.<sup>33</sup> The market wage,  $W^e$  is determined by equation (F-3) reflecting the complementarity condition as the expression uses the '=' relational operator of GAMS. This equation will be paired with the complementarity condition on the unemployment level. Equation (F-4) defines the unemployment level.

$$W_{r,l,z,t}^{res} = \chi_{r,l,z}^{rw} (1 + g_{r,t}^y)^{\omega_{r,l,z}^{rwg}} \left( \frac{1 - UE_{r,l,z,t}^z}{1 - UE_{r,l,z,t-1}^z} \right)^{\omega_{r,l,z}^{rwe}} \left( \frac{PFD_{r,h,t}}{PFD_{r,h,t-1}} \right)^{\omega_{r,l,z}^{rwp}} \quad (\text{F-2})$$

$$W_{r,l,z,t}^e \geq W_{r,l,z,t}^{res} \perp UE_{r,l,z,t}^z \geq UEMin_{r,l,z,t}^z \quad (\text{F-3})$$

$$(1 - UE_{r,l,z,t}^z) LS_{r,l,z,t}^z = LD_{r,l,z,t}^z \quad (\text{F-4})$$

The model allows for inter-sectoral wage differentials. These are represented by the  $\pi^w$  parameters that are calibrated using base year wage differentials and held fixed afterwards. This implies that all wages (for a particular skill and within a zone) move proportionately but allowing

<sup>33</sup> To deal with the possibility of zero unemployment, i.e. full employment, the reservation wage equation is formulated in terms of changes to 1 minus the unemployment rate, i.e. the rate of employment. The elasticity in this case should be positive, i.e. as we approach full employment, the reservation wage is likely to increase. The respective elasticities are linked by the formula:  $\eta = -\omega(1 - UE)/UE$ , where  $\omega$  is the elasticity with respect to the unemployment rate and  $\eta$  is the elasticity with respect to the employment rate. Note that this relation is not constant and depends on the rate of unemployment. For example, if the elasticity between the reservation wage and the unemployment rate is -0.5 and the initial unemployment rate is 10 percent, the elasticity with respect to the employment rate is 4.5 ( $= -(-0.5) \cdot 0.9/0.1$ ).

for wage differentials. Note that this assumption has potential implications for model results as any inter-sectoral shifts in labor may imply shifts in aggregate labor productivity (irrespective of technological change). Equation (F-5) sets the sectoral wage equal to the equilibrium wage (by zone) adjusted by the sector-specific wage differential.

$$W_{r,l,a} = \pi_{r,l,a}^w W_{r,l,z}^e \quad \text{for } a \in z \quad (\text{F-5})$$

If the rural to urban migration module is active, it depends on the so-called urban premium, i.e. the extent to which the urban wage exceeds the rural wage. Equation (F-6) defines the average wage in each of the zones,  $W^a$  (by skill). Equation (F-7) defines the urban premium,  $\pi^{Urb}$ . It is the ratio of the expected urban wage to the expected rural wage, where the actual average wage is adjusted by the probability of employment. Migration, which will be a function of the urban premium, is described below as part of the dynamics of the model. In the comparative static version of the model, it is assumed that labor supply is fixed (by zone).

$$W_{r,l,z}^a = \frac{\sum_{a \in z} W_{r,l,a} L_{r,l,a}^d}{\sum_{a \in z} L_{r,l,a}^d} \quad (\text{F-6})$$

$$\pi_{r,l}^{Urb} = \frac{(1 - UE_{r,l,Urb}^z) W_{r,l,Urb}^a}{(1 - UE_{r,l,Rur}^z) W_{r,l,Rur}^a} \quad (\text{F-7})$$

Another potentially useful concept is the notion of the skill premium, i.e. the extent to which skilled wages exceed unskilled wages.<sup>34</sup> Equation (F-8) defines the average economy-wide wage for each skill type,  $W^t$ .<sup>35</sup> Equation (F-9) defines the so-called skill premium,  $\pi^s$ . It is the percentage by which the reference wage exceeds other wages, where the reference wage is equal to a weighted average of wages over a subset of labor skills (where the subset is indexed by  $lr$ )—the subset is typically the set of skilled labor types.

$$W_{r,l}^t = \frac{\sum_a W_{r,l,a} L_{r,l,a}^d}{\sum_a L_{r,l,a}^d} \quad (\text{F-8})$$

$$\pi_{r,l,t}^s = \frac{\sum_{lr} W_{r,lr,t} LAB_{r,lr,t}^s / \sum_{lr} LAB_{r,lr,t}^s}{W_{r,l,t}^t} - 1 \quad (\text{F-9})$$

The final two sets of equations define the total labor supply by skill, equation (F-10), and the total labor supply, equation (F-11), respectively,  $L^s$  and  $TL^s$ . In comparative static simulations,  $LS^z$  is fixed by zone. In dynamic simulations,  $LS^z$  grows at some 'natural' rate of growth, but may also be adjusted by rural to urban migration. This is further described below.

$$L_{r,l}^s = \sum_z LS_{r,l,z}^z \quad (\text{F-10})$$

$$TL_r^s = \sum_l L_{r,l}^s \quad (\text{F-11})$$

<sup>34</sup> In a dynamic calibration scenario, assumptions regarding the evolution of the skill premium can be used to calibrate the relative growth of skilled vs. unskilled labor or to calibrate productivity differentials across skills.

<sup>35</sup> In the case of non-segmented markets,  $W_{r,l}^t$  should be equal to  $W_{r,l,nsq}^a$ .

### 3.8.2 Capital market

The model can operate in recursive dynamic mode or in comparative static mode. Capital market closure differs in each. In the comparative static mode, there is a single vintage and aggregate capital is allocated across activities using a CET transformation function. The latter allows two extreme assumptions—fixed capital per activity, i.e. no mobility, and perfect mobility and a uniform rates of return across activities. In dynamic mode, with vintage capital, *New* capital is fully mobile across sectors. *Old* capital is released on the secondary capital market if installed capital exceeds demand, for example in declining sectors.

#### Comparative static closure

A CET transformation function is used to allocate aggregate capital in the comparative static version of the model. Equation (F-12) describes the supply of capital to activity  $a$ ,  $K^s$ , where  $\omega^k$  determines the degree of mobility across activities.<sup>36</sup> With perfect mobility, the rate of return across activities is uniform. Equation (F-13) determines the aggregate rate of return to capital,  $TR$ . Equation (F-14) is the equilibrium condition for capital—supply of capital to each activity is equal to demand. In the model implementation the equilibrium condition is substituted out.

$$\begin{cases} K_{r,a,v}^s = \gamma_{r,a,v}^k \left( \frac{PK_{r,a,v}}{TR_r} \right)^{\omega_r^k} TK_r^s & \text{if } \omega_r^k \neq \infty \\ PK_{r,a,v} = TR_r & \text{if } \omega_r^k = \infty \end{cases} \quad (\text{F-12})$$

$$TR_r TK_r^s = \sum_a \sum_v PK_{r,a,v} K_{r,a,v}^s \quad (\text{F-13})$$

$$K_{r,a,v}^s = K_{r,a,v}^v \quad (\text{F-14})$$

#### Vintage capital closure

Equation (F-15) represents the contemporaneous capital-output ratio for the *Old* vintage,  $kxRat$ . Equation (F-16) reflects a complementarity relation. On the right hand side of the first inequality is the amount of capital needed to produce aggregate output,  $XP$ . If the level of installed capital,  $K^0$ , is insufficient to produce aggregate output, the activity is in expansion and will request *New* capital. In this case, the relative rate of return,  $RR$ , i.e. the ratio of returns to *Old* capital relative to *New* capital, is equal to 1, and installed capital at the margin receives the same rate of return as *New*. If, on the other hand, the activity is over-supplied with installed capital, it is an activity in contraction, and its installed capital will be released according to a downward sloping supply curve, with a supply elasticity given by  $\eta^I$ . Equation (F-17) determines the economy-wide rate of return,  $TR$ . Equation (F-18) determines the sector- and vintage-specific rate of return. If the activity uses *New* capital,  $RR$  is equal to 1 and both *Old* and *New* capital obtain the rate of return  $TR$ . If the sector is in decline, it receives a rate of return lower than the economy-wide rate of return.

$$kxRat_{r,a,Old} = \frac{K_{r,a,Old}^v}{XPv_{r,a,Old}} \quad (\text{F-15})$$

$$K_{r,a}^0 RR_{r,a}^{\eta^I} \leq kxRat_{r,a,Old} XP_{r,a} \perp RR_{r,a} \leq 1 \quad (\text{F-16})$$

<sup>36</sup> The vintage index is for convenience only. In the comparative static version there is only a single vintage.



$$TK_r^s = \sum_a \sum_v K_{r,a,v}^v \quad (\text{F-17})$$

$$PK_{r,a,v} = RR_{r,a} TR_r \quad (\text{F-18})$$

### Determination of output by vintage

Aggregate output,  $XP$ , is determined by equilibrium conditions. Equation (F-19) determines output with *Old* capital. If the activity is in expansion,  $RR$  is equal to 1, and output produced with *Old* capital is simply the output that can be produced with installed capital. If the activity is in contraction, the right-hand term is the supply of *Old* capital in the activity. Equation (F-20) ensures that the sum of output across vintages equals aggregate output. In the vintage capital version of the model, it essentially determines production with *New* capital. In the comparative static version, it trivially equates total production to production produced by the only vintage that is labeled *Old*.

$$kxRat_{r,a,Old} XPv_{r,a,Old} = K_{r,a}^0 RR_{r,a}^{\eta_r^l} \quad (\text{F-19})$$

$$XP_{r,a} = \sum_v XPv_{r,a,v} \quad (\text{F-20})$$

### 3.8.3 Land market

The supply side of the land market has two components. The first component provides the aggregate supply of land. The second step allocates aggregate land across different activities allowing for a nested CET structure, the possibility of perfect mobility, and the use of the adjusted CET that preserves land additivity.

The aggregate land supply curve is allowed to have four shapes—constant elasticity, a logistic curve with an upward asymptote, a generalized hyperbola also with an upward asymptote, and perfectly horizontal. The aggregate land supply curve is given by equation (F-21). The initial elasticity of aggregate land is given by  $\eta^t$ .

$$\left\{ \begin{array}{ll} TLand_r = \chi_r^t \left( \frac{PTLand_r}{PGDP_r} \right)^{\eta_r^t} & \text{iso-elastic} \\ TLand_r = \frac{TLand_r^{Max}}{1 + \chi_r^t e^{-\gamma_r^t (PTLand_r/PGDP_r)}} & \text{logistic} \\ TLand_r = TLand_r^{Max} - \chi_r^t \left( \frac{PTLand_r}{PGDP_r} \right)^{-\gamma_r^t} & \text{hyperbola} \\ PTLand_r = PGDP_r & \text{if } \eta_r^t = \infty \end{array} \right. \quad (\text{F-21})$$

The iso-elastic curve by definition has a constant elasticity given by  $\eta^t$ . The logistic function, in a simplified form is given by:

$$S = \frac{\bar{S}}{1 + \chi e^{-\gamma P}}$$

where  $S$  is supply, the land maximum is given by  $\bar{S}$  and  $P$  is the real price of land. The supply elasticity is given by the following formula:

$$\eta = \gamma P \left( \frac{\bar{S} - S}{\bar{S}} \right)$$

The  $\gamma$  coefficient can be calibrated using the expression above given initial values for  $\eta$ ,  $S$ ,  $\bar{S}$  and  $P$ . The calibration formula is given by:

$$\gamma = \frac{\eta}{P} \left( \frac{\bar{S}}{\bar{S} - S} \right)$$

Calibration of the  $\chi$  parameter is given by the following expression and requires first calibrating the  $\gamma$  parameter:

$$\chi = e^{\gamma P} \left( \frac{\bar{S} - S}{\bar{S}} \right)$$

The generalized hyperbola, using the same notation is given by:

$$S = \bar{S} - \chi P^{-\gamma}$$

The supply elasticity is given by the following formula:

$$\eta = \gamma \left( \frac{\bar{S} - S}{S} \right)$$

The  $\gamma$  coefficient can be calibrated using the expression above given initial values for  $\eta$ ,  $S$ ,  $\bar{S}$  and  $P$ . The calibration formula is given by:

$$\gamma = \eta \left( \frac{S}{\bar{S} - S} \right)$$

Calibration of the  $\chi$  parameter is given by the following expression and requires first calibrating the  $\gamma$  parameter:

$$\chi = P^\gamma (\bar{S} - S)$$

The user chooses the functional form by setting the global parameter named **TASS** towards the top of a simulation file. Valid options are **KELAS** for the iso-elastic form, **LOGIST** for the logistic specification, **HYPERB** for the generalized hyperbola and **INFY** for the horizontal supply function.<sup>37</sup>

Total land is allocated across different uses according to the relative rates of return across uses. A nested CET structure is implemented that allows for a degree of user control.<sup>38</sup> Figure 10 provides a graphic depiction of the land allocation specification. The number of land bundles (*XLB*) is under user control, but typically there should be at least one. The first land bundle is in the top CET nest with an intermediate land bundle (*XLB*). All other land bundles are nested within the intermediate land bundle.<sup>39</sup> It is possible to mimic the GTAP standard nesting, a single level nest,

<sup>37</sup> These options are defined in the `model.gms` file with a set named `landAssumptions`. The subset named `tass` contains the user-chosen specification.

<sup>38</sup> The implementation was first developed to mimic the CET structure of the land allocation specification in the MAGNET model.

<sup>39</sup> To mimic MAGNET's specification, `v-f`, `ocr-a`, `oap-a` are mapped to land bundle 1, `sug`, `ctl` and `rnk` are mapped to land bundle 2 and `ric`, `wht`, `gro` and `osd` are mapped to land bundle 2. The MAGNET model has two intermediate bundles and only three elasticities. In this specification, there are four intermediate bundles. Thus the top elasticity and the elasticity for land bundle 1 (horticulture, etc.), in MAGNET, are identical (and

by defining only one land bundle and use the GTAP transformation elasticity for that bundle. The top level elasticity is essentially irrelevant since in this case the intermediate land bundle would not be used.

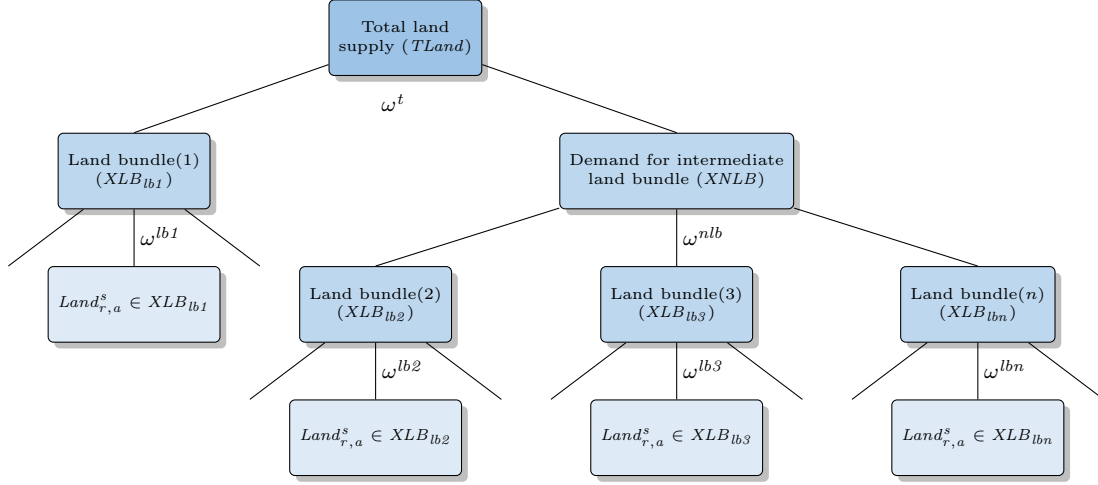


Figure 10: **CET nest for land allocation**

The top level nest decomposes total land supply into land bundle 1 and the intermediate land bundle. Equations (F-22) and (F-23) provide the first order conditions and hold irrespective of the CET specification used—either the standard, with `ifLandCET` set to 1, or the adjusted, volume preserving, CET specification. Equation (F-24) defines the aggregate price index of land,  $PTLandN$ . The index expression depends on the choice of CET specification. For the standard CET, the price index will be identical to the average price (defined next). For the adjusted CET, the price index will not generally be equal to the average price. Note that the price index expressions are not incorporated in the model under the assumption of perfect transformation, i.e. when the transformation elasticity is infinite. The average price of aggregate land,  $PTLand$  is given in equation (F-25) and holds for both versions of the CET.

$$\begin{cases} XLB_{r,lb1} = \gamma_{r,lb1}^{lb} \left( \frac{PLB_{r,lb1}}{PTLandN_r} \right)^{\omega_r^t} TLand_r & \text{if } \omega_r^t \neq \infty \\ PLB_{r,lb1} = PTLand_r & \text{if } \omega_r^t = \infty \end{cases} \quad (\text{F-22})$$

$$\begin{cases} XNLB_r = \gamma_r^{nlb} \left( \frac{PNLB_r}{PTLandN_r} \right)^{\omega_r^t} TLand_r & \text{if } \omega_r^t \neq \infty \\ PNLB_r = PTLand_r & \text{if } \omega_r^t = \infty \end{cases} \quad (\text{F-23})$$

$$\begin{cases} PTLandN_r = \left[ \gamma_{r,lb1}^{lb} PLB_{r,lb1}^{1+\omega_r^t} + \gamma_r^{nlb} PNLB_r^{1+\omega_r^t} \right]^{\frac{1}{1+\omega_r^t}} & \text{if } ifLandCET \\ PTLandN_r = \left[ \gamma_{r,lb1}^{lb} PLB_{r,lb1}^{\omega_r^t} + \gamma_r^{nlb} PNLB_r^{\omega_r^t} \right]^{\frac{1}{\omega_r^t}} & \text{if } \neg ifLandCET \end{cases} \quad (\text{F-24})$$

$$PTLand_r TLand_r = PLB_{r,lb1} XLB_{r,lb1} + PNLB_r XNLB_r \quad (\text{F-25})$$

set to  $\sigma^1$  in the MAGNET documentation, see <http://www.magnet-model.org/MagnetModuleDescription.pdf>, page 74). The elasticity for the intermediate land bundle and the second land bundle (pasture, etc.) would also be identical (and set to  $\sigma^2$ ). And the third land bundle (cereals) would have the elasticity given by  $\sigma^3$ .

The second level nest decomposes the intermediate land bundle (if it is defined) into a number of user determined land bundles, excluding the first land bundle. Equation (F-26) defines the demand for the land bundles as a share of the intermediate land bundle.<sup>40</sup> The price index of the intermediate land bundle is defined in equation (F-27), allowing for both CET specifications and is only included if the transformation elasticity at this nest is finite. The average price of the land bundle is defined in equation (F-28).

$$\begin{cases} XLB_{r,lb} = \gamma_{r,lb}^{lb} \left( \frac{PLB_{r,lb}}{PNLBN_r} \right)^{\omega_r^{nlb}} XNLB_r & \text{if } \omega_r^{nlb} \neq \infty \\ PLB_{r,lb} = PNLB_r & \text{if } \omega_r^{nlb} = \infty \end{cases} \quad (\text{F-26})$$

$$\begin{cases} PNLBN_r = \left[ \sum_{\{lb:lb \neq lb1\}} \gamma_{r,lb}^{lb} PLB_{r,lb}^{1+\omega_r^{nlb}} \right]^{\frac{1}{1+\omega_r^{nlb}}} & \text{if } ifLandCET \\ PNLBN_r = \left[ \sum_{\{lb:lb \neq lb1\}} \gamma_{r,lb}^{lb} PLB_{r,lb}^{\omega_r^{nlb}} \right]^{\frac{1}{\omega_r^{nlb}}} & \text{if } \neg ifLandCET \end{cases} \quad (\text{F-27})$$

$$PNLB_r XNLB_r = \sum_{\{lb:lb \neq lb1\}} PLB_{r,lb} XLB_{r,lb} \quad (\text{F-28})$$

The bottom land nest(s) decompose each of the land bundles into the primary land allocation to each activity  $a$ —each activity  $a$  (that uses land) must be mapped to one, and only one, land bundle. Equation (F-29) defines the supply of land to activity  $a$  that has been mapped to land bundle  $lb$ . Equation (F-30) defines the price index for the bundles that depends on the CET specification. And equation (F-31) defines the average price of the land bundles.

$$\begin{cases} Land_{r,a}^s = \gamma_{r,a}^t \left( \frac{PLand_{r,a}}{PLBN_{r,lb}} \right)^{\omega_{r,lb}^{lb}} XLB_{r,lb} & \text{if } \omega_{r,lb}^{lb} \neq \infty \quad a \in \{lb\} \\ PLand_{r,a} = PLB_{r,lb} & \text{if } \omega_{r,lb}^{lb} = \infty \end{cases} \quad (\text{F-29})$$

$$\begin{cases} PLBN_{r,lb} = \left[ \sum_{a \in \{lb\}} \gamma_{r,a}^t PLand_{r,a}^{1+\omega_{r,lb}^{lb}} \right]^{\frac{1}{1+\omega_{r,lb}^{lb}}} & \text{if } ifLandCET \\ PLBN_{r,lb} = \left[ \sum_{a \in \{lb\}} \gamma_{r,a}^t PLand_{r,a}^{\omega_{r,lb}^{lb}} \right]^{\frac{1}{\omega_{r,lb}^{lb}}} & \text{if } \neg ifLandCET \end{cases} \quad (\text{F-30})$$

$$PLB_{r,lb} XLB_{r,lb} = \sum_{a \in \{lb\}} PLand_{r,a} Land_{r,a}^s \quad (\text{F-31})$$

The final equation in the land market module, equation (F-32) equates land supply to activity  $a$  to its respective demand thus determining the equilibrium and sector-specific price for land,  $PLand$ . The equation is substituted out of the model specification.

$$Land_{r,a}^s = Land_{r,a}^d \quad (\text{F-32})$$

<sup>40</sup> The index  $lb$  excludes the first land bundle.

### 3.8.4 Market for natural resources

Natural resources are assumed to be activity-specific, for example crude oil reserves. The supply of natural resources,  $XNRF^s$ , is governed by a kinked iso-elastic supply curve. The model allows the supply response to differ according to whether market conditions are leading to increased supply, the "hi" elasticity, versus a contracting market, the "lo" elasticity. Ideally, we could implement this with a regime shift using continuous functions, but discontinuous derivatives. Instead, we add an equation to the model that determines the supply elasticity using the sigmoid function, also known as a logistic function.<sup>41</sup> The sigmoid function is defined by:

$$f(x) = \frac{1}{1 + e^{-x}}$$

Equation (F-33) defines the applied elasticity as a function of market conditions. The user enters the 'lo' and 'hi' elasticities in the parameters  $\eta^{nrfx}$ . One would typically assume that the 'lo' elasticity is higher than the 'hi' elasticity, i.e. it is harder to expand output than to contract output. If the two are identical, the equation is trivial. The argument in the sigmoid function reflects market growth and will be positive for expansion and negative for contraction. The  $\kappa$  parameter captures the 'speed' of adjustment between the 'lo' and 'hi' elasticities. The greater the value, the more rapid the transition. We have used a value of 30 with few numerical problems.<sup>42</sup>

Equation (F-34) reflects the supply function for the natural resource. For finite elasticities, it is an iso-elastic function, however, where the supply elasticity is determined by market conditions as described above. It is a function of the growth of the real price of the natural resource and the supply curve shifts according to lagged supply.<sup>43</sup> The (finite) supply curve has two shifters that can be used to target either the regional output price of the commodity being produced or the global price. To target the regional price, exogenize the output price,  $PP$ , and endogenize the regional shifter,  $\chi^{nrf}$ . To target the global price, exogenize the average producer price,  $PW$ ,<sup>44</sup> and endogenize the global shifter,  $\chi^{w,nrf}$ . The supply curve can also be horizontal, in which case the supply price is fixed relative to an anchor price, herein the GDP deflator. Equation (F-35) determines the equilibrium price. The latter equation is substituted out of the model.

$$\eta_{r,a,t}^{nrf} = \eta_{r,a,lo}^{nrfx} + \frac{\eta_{r,a,hi}^{nrfx} - \eta_{r,a,lo}^{nrfx}}{1 + \exp\left(-\kappa \left[\frac{XNRF_{r,a,t}^d}{XNRF_{r,a,t-1}^d} - 1\right]\right)} \quad (\text{F-33})$$

$$\begin{cases} XNRF_{r,a,t}^s = \chi_a^{w,nrf} \chi_{r,a}^{nrf} XNRF_{r,a,t-1}^s \left( \frac{\chi_{r,a}^{nrfp} PNR_{r,a,t} / PGDPMP_{r,t}}{PNR_{r,a,t-1} / PGDPMP_{r,t-1}} \right)^{\eta_{r,a}^{nrf}} & \text{if } \eta_{r,a}^{nrf} \neq \infty \\ \chi_{r,a}^{nrfp} PNR_{r,a} = PGDPMP_r & \text{if } \eta_{r,a}^{nrf} = \infty \end{cases} \quad (\text{F-34})$$

$$XNRF_{r,a}^s = XNRF_{r,a}^d \quad (\text{F-35})$$

<sup>41</sup> This is used for example in the CAPRI model of global agriculture with a focus on the EU, Britz et al. (2007).

<sup>42</sup> The sigmoid function is an intrinsic function in GAMS.

<sup>43</sup> This is somewhat different from the previous specification where the supply shifter was constant unless endogenous for calibration purposes.

<sup>44</sup> Defined in equation (M-23).

### 3.8.5 Water market

The market for water is somewhat similar to the land market. An aggregate supply curve is provided—and aggregate supply is then allocated to different uses using a nested CET structure. However, unlike land, water is in direct use in only the irrigated crop sectors. Demand for water in other sectors is specified using aggregate demand functions.

The aggregate water supply curve is allowed to have four shapes—constant elasticity, a logistic curve with an upward asymptote, a generalized hyperbola also with an upward asymptote, and perfectly horizontal. The aggregate water supply curve is given by equation (F-36). The initial elasticity of aggregate water is given by  $\eta^w$ .

$$\left\{ \begin{array}{ll} TH2O_r = \chi_r^{h2o} \left( \frac{PTH2O_r}{PGDP_r} \right)^{\eta_r^w} & \text{iso-elastic} \\ TH2O_r = \frac{TH2O_r^{Max}}{1 + \chi_r^{h2o} e^{-\gamma_r^{tw} (PTH2O_r/PGDP_r)}} & \text{logistic} \\ TH2O_r = TH2O_r^{Max} - \chi_r^{h2o} \left( \frac{PTH2O_r}{PGDP_r} \right)^{-\gamma_r^{tw}} & \text{hyperbola} \\ PTH2O_r = PGDP_r & \text{if } \eta_r^w = \infty \end{array} \right. \quad (\text{F-36})$$

The iso-elastic curve by definition has a constant elasticity given by  $\eta^w$ . The calibration and other formulas for the logistic and hyperbola functions are the same as the land market expressions and are not repeated here.

Water use at the sub-aggregate level is represented by a number of 'bundles',  $H2OBnd$ . Table 3.4 lists the bundles and their relationships. The marketed water bundles are allocated in two CET nests. The top nest allocates water between agriculture and non-agriculture. These bundles are indexed by  $wb1$ . The second level nests include crops and livestock linked to the agriculture bundle, and industrial and municipal water use linked to the non-agriculture bundle. The second level bundles are indexed by  $wb2$ . The second level bundles are also segmented into two groups— $wba$  and  $wbi$ . The former allocates water across different activities—for example irrigated crops—and the latter is linked to an aggregate water demand—not explicitly linked to specific activities. The set  $wbx$  refers to exogenous water demand.

Table 3.4: Aggregate water bundles

Name	Description	Set	Dependency	Type
ENV	Environmental services		$TH2O$	$wbx$
GRD	Groundwater recharge		$TH2O$	$wbx$
AGR	All agriculture	$wb1$	$TH2O^m$	
CRP	Crops	$wb2$	AGR	$wba$
LVS	Livestock	$wb2$	AGR	$wbi$
NAG	Not agriculture	$wb1$	$TH2O^m$	
IND	Industrial water use	$wb2$	NAG	$wbi$
MUN	Municipal water use	$wb2$	NAG	$wbi$

At the top level water supply is allocated between marketed water and non-marketed water. The latter is exogenous and split into two variables—water for environmental services and water required for ground water recharge. In equilibrium, one would assume the latter is close to zero, i.e. ground water withdrawal closely matches ground water extraction. In the current database, both

of these exogenous water uses are initially set at zero. A policy shock may include making these positive in a future time period—perhaps permanently for environmental services and temporarily for ground water recharge. Marketed water is then the difference between total water supply and exogenous demand. Equation (F-37) is the market supply equilibrium equation equating total water supply to marketed water plus the sum of all exogenous water demand. The variable  $H2O^m$  represents marketed water.

$$TH2O_r = TH2O_r^m + \sum_{wb \in wbx} H2OBnd_{r,wb} \quad (\text{F-37})$$

Marketed water is allocated across the various water bundles using a nested (adjusted) CET specification.<sup>45</sup> Figure 11 illustrates the nested structure.

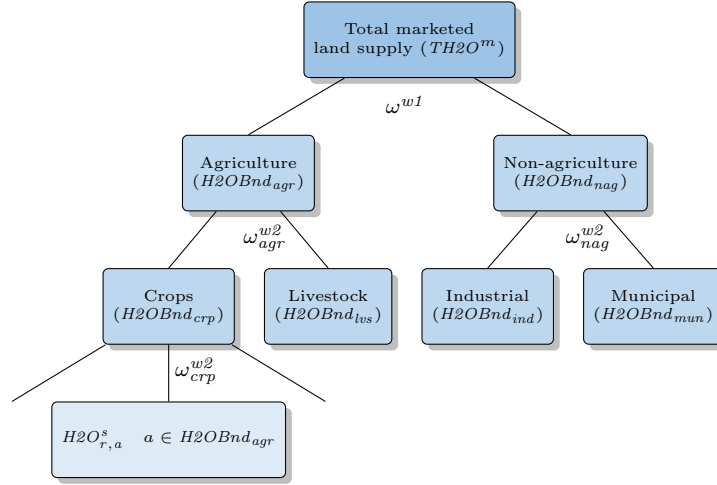


Figure 11: CET nest for water allocation

The first level nest allocates aggregate marketed water to the agricultural and non-agricultural bundles using an adjusted CET specification. The supply expressions are given in equation (F-38) that allows for perfect transformation. With reference to Table the index  $wb1$  covers the AGR and NAG water bundles.<sup>46</sup> If the transformation elasticity is finite, equation (F-39) provides the expression for the CET aggregate price index for  $TH2O^m$ . The aggregate water supply price is given by equation (F-40).

$$\begin{cases} H2OBnd_{r,wb1} = \gamma_{r,wb1}^{w1} \left( \frac{PH2OBnd_{r,wb1}}{PTH2O_r^n} \right)^{\omega_r^{w1}} TH2O_r^m & \text{if } \omega_r^{w1} \neq \infty \\ PH2OBnd_{r,wb1} = PTH2O_r & \text{if } \omega_r^{w1} = \infty \end{cases} \quad (\text{F-38})$$

$$PTH2O_r^n = \left[ \sum_{wb1} \gamma_{r,wb1}^{w1} PH2OBnd_{r,wb1}^{\omega_r^{w1}} \right]^{1/\omega_r^{w1}} \quad \text{if } \omega_r^{w1} \neq \infty \quad (\text{F-39})$$

$$PTH2O_r TH2O_r^m = \sum_{wb1} PH2OBnd_{r,wb1} H2OBnd_{r,wb1} \quad (\text{F-40})$$

<sup>45</sup> The adjusted CET specification preserves quantity additivity—see Appendix A.

<sup>46</sup> The GAMS code is written in a relatively flexible way so that the user can largely change the set definitions for all water bundles and the relevant subsets and mappings.

The next nest decomposes the AGR and LVS bundles into crops (CRP) and livestock (LVS) on the one hand and industrial (IND) and municipal (MUN) water demand on the other hand. Equation (F-41) determines the supply for the second level bundles—indexed by  $wb2$ . Each of the  $wb2$  bundles is mapped to one, and only one, top level bundle indexed by  $wb1$ . Equation (F-42) defines the relevant aggregate price index that is used to determine the allocation. It is only used in the case of a finite transformation elasticity. Equation (F-43) defines the price of the aggregate  $wb1$  bundles, i.e. the price of the AGR and NAG bundles.

$$\begin{cases} H2OBnd_{r,wb2} = \gamma_{r,wb2}^{w2} \left( \frac{PH2OBnd_{r,wb2}}{PH2OBnd_{r,wb1}^n} \right)^{\omega_{r,wb1}^{w2}} H2OBnd_{r,wb1} & \text{if } \omega_{r,wb1}^{w2} \neq \infty \\ PH2OBnd_{r,wb2} = PH2OBnd_{r,wb1} & \text{if } \omega_{r,wb1}^{w2} = \infty \end{cases} \quad \text{wb2} \in \text{wb1} \quad (\text{F-41})$$

$$PH2OBnd_{r,wb1}^n = \left[ \sum_{wb2 \in wb1} \gamma_{r,wb2}^{w2} PH2OBnd_{r,wb2}^{\omega_{r,wb1}^{w2}} \right]^{1/\omega_{r,wb1}^{w2}} \quad \text{if } \omega_{r,wb1}^{w2} \neq \infty \quad (\text{F-42})$$

$$PH2OBnd_{r,wb1} H2OBnd_{r,wb1} = \sum_{wb2 \in wb1} PH2OBnd_{r,wb2} H2OBnd_{r,wb2} \quad (\text{F-43})$$

The second level bundles, indexed by  $wb2$ , are segmented into two subsets. The first subset, indexed by  $wba$ , has an additional CET nest that maps the aggregate supply to individual production activities ( $a$ ). With the current version of the database, the AGR bundle is the only bundle that maps to water demand by production activities. The water bundles in the second subset, indexed by  $wbi$ , are set equal to an aggregate demand for water over a relevant subset of activities mapped to that bundle.<sup>47</sup>

Equation (F-44) determines the supply of water to each activity  $a$  that is mapped to the aggregate water supply indexed by  $wba$  (currently limited to irrigated crops mapped to the AGR bundle).<sup>48</sup> Equation (F-45) defines the relevant aggregate price index that is used to determine the allocation. It is only used in the case of a finite transformation elasticity. Equation (F-46) defines the price of the aggregate  $wba$  bundle(s).

$$\begin{cases} H2O_{r,a}^s = \gamma_{r,a}^{w3} \left( \frac{PH2O_{r,a}}{PH2OBnd_{r,wba}^n} \right)^{\omega_{r,wba}^{w2}} H2OBnd_{r,wba} & \text{if } \omega_{r,wba}^{w2} \neq \infty \\ PH2O_{r,a} = PH2OBnd_{r,wba} & \text{if } \omega_{r,wba}^{w2} = \infty \end{cases} \quad a \in wba \quad (\text{F-44})$$

$$PH2OBnd_{r,wba}^n = \left[ \sum_{a \in wba} \gamma_{r,a}^{w3} PH2O_{r,a}^{\omega_{r,wba}^{w2}} \right]^{1/\omega_{r,wba}^{w2}} \quad \text{if } \omega_{r,wba}^{w2} \neq \infty \quad (\text{F-45})$$

$$PH2OBnd_{r,wba} H2OBnd_{r,wba} = \sum_{a \in wba} PH2O_{r,wba} H2O_{r,a}^s \quad (\text{F-46})$$

Equation (F-47) determines the equilibrium market price of water,  $PH2O$ , delivered to activity  $a$ . Water demand is provided by equation (P-37). The equilibrium condition is substituted out of the model and the variables  $H2O^s$  and  $H2O^d$  are replaced with  $H2O$ .

$$H2O_{r,a}^s = H2O_{r,a}^d \quad (\text{F-47})$$

<sup>47</sup> This is an ad hoc treatment of water demand that generates an equilibrium price. However, the demand for water in the relevant production activities do not form part of the cost structure of these activities—unlike the case for irrigated crops.

<sup>48</sup> The same elasticity parameter,  $\omega^{w2}$  is used for all of the aggregate water bundles, irrespective of the nest level.



For the aggregate water bundles indexed by  $wbi$ , demand is constructed as a function of the aggregate production volume of the constituent activities and includes a price sensitive component. Equation (F-48) defines demand for aggregate water,  $H2OBnd^d$ . Demand is a function of a volume index over the activities mapped to the water bundles—subject to an elasticity given by  $\eta^{h2ob}$ . In addition, demand responds to the real price of the relevant bundle. Equation (F-49) determines the equilibrium price for the aggregate bundle. The equilibrium expression is substituted out of the model.

$$H2OBnd_{r,wbi}^d = \frac{\alpha_{r,wbi}^{h2ob}}{\lambda_{r,wbi}^{h2ob}} \left( \frac{PH2OBnd_{r,wbi}^d}{PGDPMPr} \right)^{-\epsilon_{r,wbi}^{h2ob}} \left[ \frac{\sum_{a \in wbi} PP_{r,a,0} XP_{r,a}}{\sum_{a \in wbi} PP_{r,a,0} XP_{r,a,0}} \right]^{\eta_{r,wbi}^{h2ob}} \quad (F-48)$$

$$H2OBnd_{r,wbi} = H2OBnd_{r,wbi}^d \quad (F-49)$$

### 3.8.6 Factor taxes and prices

Producers pay the market price for factors adjusted by factor taxes. Equations (F-50), (F-51), (F-52), (F-53) and (F-54) determine the producer prices of the factors of production, respectively for labor, capital, land, natural resources and water.

$$W_{r,l,a}^p = (1 + \tau_{r,l,a}^l) W_{r,l,a} \quad (F-50)$$

$$PK_{r,a,v}^p = (1 + \tau_{r,a,v}^k) PK_{r,a,v} \quad (F-51)$$

$$PLand_{r,a}^p = (1 + \tau_{r,a}^t) PLand_{r,a} \quad (F-52)$$

$$PNRF_{r,a}^p = (1 + \tau_{r,a}^n) PNRF_{r,a} \quad (F-53)$$

$$PH2O_{r,a}^p = (1 + \tau_{r,a}^w) PH2O_{r,a} \quad (F-54)$$

## 3.9 National accounts and model closure

### 3.9.1 National accounts

Definition of the national accounts would normally be part of the post-processing portion of a simulation, i.e. it is not necessarily part of the core definition of a CGE model. However, when running the model in dynamics, it is typically useful to have some of the key national accounting identities available for targeting purposes, for example to target the growth of per capita GDP, or a macro ratio such as the share of government or investment expenditures relative to GDP.

Equations (M-1) and (M-2) define respectively  $PFd$  and  $YFD$  for households. In essence  $PFd_h$  is the consumer price index (CPI) and  $YFD_h$  is personal consumption expenditures (PCE) in nominal terms.

$$PFd_{r,h} = \chi_{r,h} \sum_i \varphi_{r,i,h}^a PA_{r,i,h}^a \quad (M-1)$$

$$YFD_{r,h} = \sum_i PA_{r,i,h}^a XA_{r,i,h} \quad (\text{M-2})$$

Equation (M-3) defines nominal GDP at market price,  $GDPMP_r$ , using the standard national accounting identity, i.e. it is the sum of domestic absorption, plus the exports of international trade and transport services and the net trade balance for all other goods and services evaluated at their respective border prices. Equation (M-4) defines real GDP at market price evaluated at base year prices,  $RGDPMP_r$ . Equation (M-5) defines the GDP at market price deflator,  $PGDPMP_r$ . Equation (M-6) defines real per capita GDP,  $RGDPpc_r$ . Equation (M-7) defines the growth in real per capita GDP,  $g^y$ . In comparative static exercises, it measures the difference in real per capita GDP between the baseline and the shock simulation.<sup>49</sup> In dynamic simulations the equation allows for multi-year time steps as measured by  $n$ . In most dynamic baseline or reference simulations, the growth rate of real per capita GDP will be exogenous and an economy-wide variable, for example a uniform labor productivity shifter, is endogenous and will serve as the instrument to target GDP growth. In policy or alternative simulations, the economy-wide factor would normally be exogenous and the growth rate of GDP would be endogenous.

$$\begin{aligned} GDPMP_r = & \sum_{fd} YFD_{r,fd} + \sum_m PD_{r,m} XTT_{r,m} \\ & + \sum_i \left\{ \sum_d PWE_{r,i,d} XW_{r,i,d}^s - \sum_s PWM_{s,i,r} XW_{s,i,r}^d \right\} \end{aligned} \quad (\text{M-3})$$

$$\begin{aligned} RGDPMP_r = & \sum_{fd} PFD_{r,fd,0} XFD_{r,fd} + \sum_m PD_{r,m,0} XTT_{r,m} \\ & + \sum_i \left\{ \sum_d PWE_{r,i,d,0} XW_{r,i,d}^s - \sum_s PWM_{s,i,r,0} XW_{s,i,r}^d \right\} \end{aligned} \quad (\text{M-4})$$

$$PGDPMP_r RGDPMP_r = GDPMP_r \quad (\text{M-5})$$

$$RGDPpc_r = \frac{RGDPMP_r}{Pop_r} \quad (\text{M-6})$$

$$RGDPpc_{r,t} = (1 + g_{r,t}^y)^n RGDPpc_{r,t-n} \quad (\text{M-7})$$

### 3.9.2 Closure

#### Government accounts

Equation (M-8) defines nominal government savings,  $S^g$ , as the difference between government revenues and government's current expenditures on goods and services. When an emissions quota regime is put into place, revenues from the quotas<sup>50</sup> accrue to the government and eventually to households using the default closure with a lump sum tax on households. Note that the counterpart account of the quota income appears in the balance of payments expression, i.e. as a flow from the rest of the world. Equation (M-8) defines real government savings,  $RS^g$ . In the standard closure, the public deficit is held fixed in real terms (in order to preserve price homogeneity). The direct

<sup>49</sup> The value of  $n$  is always 1 in comparative static simulations.

<sup>50</sup> Revenues are positive if a region is a net seller and negative if a region is a net buyer.

tax rate,  $\kappa^h$ , adjusts to the target for the public deficit. Because there is a single household, this is equivalent to a change in lump sum taxes.

$$S_r^g = \sum_{gy} YGOV_{r,gy} + \sum_{em} Quota Y_{r,em}^{Emi} - YFD_{r,gov} \quad (M-8)$$

$$RS_r^g = \frac{S_r^g}{PGDPM P_r} \quad (M-9)$$

### Capital account

In the model's standard closure, foreign savings are fixed in real terms. Equation (M-10) determines  $S^f$ . For all but one region, the residual region, foreign saving is equal to a fixed level. The second part of the equation, guarantees that the sum across regions is identically zero, and thus sets the level for the residual region.<sup>51</sup>

$$\begin{cases} S_r^f = \bar{S}_r^f & \text{if } r \neq rres \\ \sum_r S_r^f \equiv 0 & \text{if } r = rres \end{cases} \quad (M-10)$$

An alternative, inspired from the GTAP model (Hertel (1997)), allows for foreign savings to adjust to relative differentials in expected rates of return—with a specific definition of what is meant by the expected rate of return.

We start with the definition of the expected end-of-period capital stock, equation (M-11). This is simply the standard motion equation for the capital stock based on this period's beginning-of-period capital stock and investment level.

$$TK_{r,t}^e = (1 - \delta_{r,t}) TK_{r,t} + XFD_{r,inv,t} \quad (M-11)$$

Equation (M-12) defines the aggregate after-tax rate of return. The factor  $\chi^k$  converts the normalized rate of return to the non-normalized rate of return.<sup>52</sup> Equation (M-13) defines the net current rate of return, the rental rate adjusted for the price of replacing capital, less the depreciation rate. Therefore, a tariff reduction, that would lower the price of investment goods, would reduce the replacement price of capital and increase the rate of return (all else equal). The expected rate of return is given by equation (M-14). Regions with relatively high investment rates, i.e, that would lead to relatively high growth in the capital stock and thus depress the capital rate of return, would tend to see a reduction in the expected rate of return. The degree of adjustment is controlled by the parameter  $\epsilon^{ror}$ .

$$R_{r,t} = \chi_r^k TR_{r,t} (1 - k_r^k) \quad (M-12)$$

$$R_{r,t}^c = \frac{R_{r,t}}{PFD_{r,inv,t}} - \delta_{r,t} \quad (M-13)$$

$$R_{r,t}^e = R_{r,t}^c \left[ \frac{TK_{r,t}^e}{TK_{r,t}} \right]^{-\epsilon^{ror}} \quad (M-14)$$

<sup>51</sup> It is best if the user makes sure that the sum of exogenous foreign savings equals 0 to avoid unintended consequences, but at least with this formulation, a small deviation will not create any problems.

<sup>52</sup> In the base year,  $TR$  is set to 1, and the non-normalized return will be a percent rate between 0 and perhaps up to 30%.

The level of foreign saving for each region will then be determined by equation (M-15), which equates the expected regional rates of return to the global average,  $R^g$ , adjusted for any risk premium that is embedded in the base year data. Equation (M-16) guarantees that global savings equals global investment and in essence determines  $R^g$ . High values for  $\epsilon^{ror}$  lead to relatively low adjustment in foreign savings. The default value for the GTAP model is 10.<sup>53</sup>

$$\pi_{r,t}^p R_{r,t}^c = R_t^g \quad (\text{M-15})$$

$$\sum_r S_r^f \equiv 0 \quad (\text{M-16})$$

Note that equations (M-12)-(M-14) are included in all versions of capital account closure. In addition, equation (M-17) is added to the model specification in the case of the fixed capital account and simply calculates  $R^g$  as a weighted average of regional expected returns, with the weights given in equation (M-18). In summary, the fixed capital account specification uses equations (M-10) through (M-14), (M-17) and (M-18), though in effect only equation (M-10) is relevant. The flexible capital account specification uses equations (M-11) through (M-16). The default specification is governed by setting the `savfFlag` to `capFix`. The flexible specification is deployed if `savfFlag` to `capFlex`.

$$R^g = \sum_r \varphi_r^r R_r^c \quad (\text{M-17})$$

$$\varphi_r^r = \frac{PFD_{r,inv} (XFD_{r,inv} - \delta_r TK_r^s)}{\sum_s PFD_{s,inv} (XFD_{s,inv} - \delta_s TK_s^s)} \quad (\text{M-18})$$

### Price anchors

Equation (M-19) defines the so-called manufactures unit value index,  $PMUV$  and it is equated to the export price index of manufactures from high-income countries. Foreign savings flows,  $S^f$ , are determined in real terms and are evaluated at the price  $PW^{sav}$ , that is set equal to  $PMUV$ , equation (M-20). The model requires at least one price to be fixed and treated as the numéraire. Equation (M-21) defines the numéraire,  $PNUM$ , to be equal as well to  $PMUV$ . Equation (M-22) is superfluous as it represents the expression of Walras' Law. It is used to evaluate the variable *Walras*, that should always be (near) 0 if the model solves and is consistent.

$$PMUV = \chi^{muv} \sum_{r \in HIC} \sum_{i \in MANU} \sum_d \varphi_{r,i,d}^w PWE_{r,i,d} \quad (\text{M-19})$$

$$PW^{sav} = PMUV \quad (\text{M-20})$$

$$PNUM = PMUV \quad (\text{M-21})$$

$$Walras = YFD_{rres,inv} - \left[ S_{rres}^h + S_{rres}^g + PW^{sav} S_{rres}^f + Depr Y_{rres} \right] \quad (\text{M-22})$$

In summary, the standard macro closure has household savings determined by the ELES demand system, government savings are fixed (in real terms) with the direct household tax schedule

<sup>53</sup> Users are urged to read the discussion in Hertel (1997), page 57.

endogenous, and foreign savings are fixed (with the real exchange rate adjusting to *ex ante* changes in the trade balance. Investment is savings determined. The model numéraire is the export price index of high-income manufactured exports.

### 3.9.3 Targeting prices

It is often the case, particularly in developing a baseline, that it is desirable (or required) to target price trends for key commodities, for example crude oil. The first step is to define what is meant by a 'world' price. The current version of the model defines the 'world' price to be the average producer price. The key reason is that typically the commodities to be targeted have a sector-specific resource base that can be adjusted to target an exogenous price.<sup>54</sup> Equation (M-23) defines the world price for activity  $a$ . The parameter  $\varphi^{pw}$  represents regional production weights initiated to  $XP_{r,a,0}/\sum_r PP_{r,a,0}$ .<sup>55</sup>

$$PW_a = \sum_r \varphi_{r,a}^{pw} PP_{r,a} \quad (\text{M-23})$$

To target a price, the variable  $PW$  needs to be exogenized and a counterpart needs to be endogenized. The most logical choice is to endogenize the shifter on the natural resource supply curve,  $\chi_a^{w,nrf}$ .

## 3.10 Greenhouse gas emissions

### 3.10.1 Emissions

The model incorporates a number of greenhouse gas (GHG) emissions, indexed by  $em$ . The standard database includes carbon emissions, methane (CH<sub>4</sub>), nitrous oxides (N<sub>2</sub>O) and an aggregate emissions bundle of fluoridated gases (F-gases). The input data for carbon is in millions of metric tons of CO<sub>2</sub> and the data for the other GHGs are both in physical units (metric tons) as well as in CO<sub>2</sub>-equivalent.<sup>56</sup> The model allows for conversion between CO<sub>2</sub>*e* and *Ce*, i.e. carbon equivalent, depending on the needs of the user.<sup>57</sup>

Emissions are generated by three sources: 1) direct consumption of a commodity; 2) factor-based emissions (e.g. capital, i.e. herds, in the livestock sectors); and 3) output or processed based emissions (e.g. methane from landfills). Carbon emissions in the current model version are only generated by the combustion of fossil fuels—coal, oil (crude and refined) and natural gas. Emissions of other GHGs can be a combination of all three sources of emissions. Equation (E-1) reflects the consumption based emissions over all Armington agents ( $aa$ ) and for all commodities ( $i$ ). Emissions are based on the consumption level multiplied by the rate of emission per unit of consumption ( $\rho^{Emi}$ ). The specification allows for both national- and agent-specific sourcing of imports. In the case of the latter, the relevant volume of consumption is  $XA$ . In the case of the former, the relevant consumption volumes are  $XD$  and  $XM$  and the emission rates are allowed to

<sup>54</sup> With a diagonal make-matrix, targeting the producer price and the commodity price would be identical.

<sup>55</sup> The weights are held fixed at base year levels, but they could be re-calibrated between years.

<sup>56</sup> The conversion from tons to CO<sub>2</sub>-equivalent uses the standard global warming potential (gwp) coefficients defined by the Intergovernmental Panel on Climate Change (IPCC).

<sup>57</sup> The `ifCEQ` flag determines the units. If set to 1, the model will use *Ce*, else CO<sub>2</sub>*e*. The main impact will be on the level of any endogenously determined carbon tax. The factor (44/12) is used to convert from *Ce* to CO<sub>2</sub>*e* in volume terms. The inverse ratio is used to convert prices and taxes.

vary across bundles.<sup>58</sup> Equations (E-2), (E-3), (E-4) and (E-5) reflect the factor based emissions for, respectively, labor, capital, land and natural resources.<sup>59</sup> Finally, equation (E-6) represents the processed based emissions.<sup>60</sup>

$$Emi_{r,em,i,aa} = \begin{cases} \chi_{em}^{Emi} \rho_{r,em,i,aa}^{Emi} XA_{r,i,aa} & \text{if } ArmFlag = 0 \\ \chi_{em}^{Emi} \rho_{r,em,i,aa}^{Emi,d} XD_{r,i,aa} + \chi_{em}^{Emi} \rho_{r,em,i,aa}^{Emi,m} XM_{r,i,aa} & \text{if } ArmFlag \neq 0 \end{cases} \quad (E-1)$$

$$Emi_{r,em,l,a} = \chi_{em}^{Emi} \rho_{r,em,l,a}^{Emi} L_{r,l,a}^d \quad (E-2)$$

$$Emi_{r,em,cap,a} = \chi_{em}^{Emi} \rho_{r,em,cap,a}^{Emi} \sum_v K_{r,a,aa,v}^v \quad (E-3)$$

$$Emi_{r,em,lnd,a} = \chi_{em}^{Emi} \rho_{r,em,lnd,a}^{Emi} Land_{r,a}^d \quad (E-4)$$

$$Emi_{r,em,nrs,a} = \chi_{em}^{Emi} \rho_{r,em,nrs,a}^{Emi} XNRF_{r,a}^d \quad (E-5)$$

$$Emi_{r,em,tot,a} = \chi_{em}^{Emi} \rho_{r,em,tot,a}^{Emi} XP_{r,a} \quad (E-6)$$

Equation (E-7) is the aggregate level of emissions for region  $r$ . It is the sum across all potential sources of emissions represented by the indices  $is$  and  $aa$ . The equation allows for a region specific level of emissions from exogenous sources—that could be used for example to introduce forest-based emissions. Equation (E-8) defines the global level of emissions, the sum of total regional emissions, and also allows for an exogenous level of emissions that is not region-specific.

$$EmiTot_{r,em} = \sum_{is} \sum_{aa} Emi_{r,em,is,aa} + EmiOth_{r,em} \quad (E-7)$$

$$EmiGbl_{em} = \sum_r EmiTot_{r,em} + EmiOthGbl_{em} \quad (E-8)$$

### 3.10.2 Emission price regimes

The model exhibits a degree of flexibility in implementing carbon price regimes. The simplest option is to set an emissions tax exogenously and all the required equations have already been described.<sup>61</sup> If the model is being solved in terms of CO<sub>2</sub>, a tax of \$x/TCO<sub>2</sub> is implemented as:

```
emiTax.fx(r,"co2",t) = cscale*x ;
```

If the model is being solved in terms of carbon, the same tax would need to be specified as:<sup>62</sup>

<sup>58</sup> The CO<sub>2</sub> emissions database distinguishes between domestic and import emissions, however the non-CO<sub>2</sub> database does not and the emission rates are assumed to be identical.

<sup>59</sup> The factor index  $fp$  is segmented into a number of subsets where  $l$  covers labor,  $cap$  reflects the capital factor,  $lnd$  reflects the land factor and  $nrs$  reflects the natural resource factor.

<sup>60</sup> There is a master set,  $is$ , that represents all of the labels of the SAM—including one for the total that is represented by the subset  $tot$ .

<sup>61</sup> This version of the model is really only set up to deal with carbon taxes. The effect of an emission tax on other GHGs is only captured through limited substitution and income effects as marginal abatement curves (MAC) have yet to be implemented.

<sup>62</sup> Given the `ifCEQ` flag, the two expressions can be combined into the following expression:

```
emiTax.fx(r,"co2",t) = cscale*x*(1$(not ifCEQ) + (44/12)$ifCEQ) ;
```

```
emiTax.fx(r,"co2",t) = cscale*x*(44/12) ;
```

The formulas would need adjustment if the exogenous carbon tax is implemented in terms of tons of carbon instead of tons of CO<sub>2</sub>. The rest of this section describes an alternative carbon price regime that relies on an emissions cap, with and without trading.

An emissions cap requires two additional equations—one that calculates the carbon tax for all regions within a coalition of regions (that may be as small as one region). The second equation maps the uniform coalition-wide carbon tax the regional carbon tax. Equation (E-9) defines the emissions cap constraint. Coalitions are represented by the index  $rq$ . This index is a subset of  $ra$  that represents a user-defined set of aggregate regions that can be used by the model (and the output routines). It normally includes all of the separate regions of the model—so that each modeled region can be its own coalition—plus a number of additional aggregate regions to which the the individual regions are mapped. Coalitions are then setup by defining the subset  $ra$  to which the regional mappings will already have been specified. The emissions cap equation then implicitly determines the coalition-wide emissions tax,  $\tau_{rq,em}^{EmiQ}$  that is compatible with the emissions constraint. The tax will be calculated for each coalition. Equation (E-10) maps the coalition-wide tax to the emissions tax of each individual region that is part of the  $rq$  coalition. In the absence of a permit trading regime, the emission tax revenues collected in each region are fully allocated to the regional government.

$$\sum_{r \in \{rq\}} EmiTot_{r,em} = \chi_{em}^{Cap} Emi_{rq,em}^{Cap} \quad (E-9)$$

$$\tau_{r,em}^{Emi} = \tau_{rq,em}^{EmiQ} \quad \text{if } r \in \{rq\} \quad (E-10)$$

The following code snippet shows how to implement a cap on emissions for two coalitions—one composed of developing countries indexed by "lmy" and one composed of developed countries indexed by "hic". The first line sets the `ifEmiCap` flag. The second line sets the `emFlag` flag to indicate a tax on carbon only. Lines 3 and 4 define the two coalitions. The fifth line sets the cap for each of the two coalitions. The caps represent a 20 percent reduction in aggregate carbon emissions relative to the level of emissions of the previous period. Assuming all regions are mapped to the two coalition regions, this will also reduce global emissions by 20 percent. The final four lines are required to endogenize the coalition-wide emission tax and the region-specific emission tax.<sup>63</sup> The latter will be uniform across all regions within the same coalition.

Listing 3.1: Example of a cap regime with two coalitions

```
1  ifEmiCap      = 1 ;
2  emFlag("co2") = 1 ;
3  rq("lmy")    = yes ;
4  rq("hic")    = yes ;
5  emiCap.fx(rq,"co2",tsim) = 0.8*sum(r$mapr(rq,r),emiTot.l(r,"co2",tsim-1)*emiTot0(r,"co2") ;
6  emiRegTax.lo(rq,"co2",tsim) = -inf ;
7  emiRegTax.up(rq,"co2",tsim) = +inf ;
8  emiTax.lo(r,"co2",tsim) = -inf ;
9  emiTax.up(r,"co2",tsim) = +inf ;
```

Emissions trading is setup by specifying regional emission quotas. The sum of the quotas within a coalition must add up to the emissions cap for the coalition. Equation (E-11) determines the value of emissions trade for each region within a coalition. It is equal to the level of the coalition-wide emission tax times the difference between the quota allocation and actual emissions. If the

<sup>63</sup> The last two lines can potentially lead to errors for more complex coalitions as it endogenizes the emissions tax for all model regions. We are assuming here that all model regions are in one of the coalitions

difference is positive, the value of trade will be positive. It is assumed that the value of the trading rights goes directly to the government (with the counterbalancing account in the omitted balance of payments constraint), see equation (M-8). The code snippet below shows how to set up a cap and trade regime at the global level. It is assumed that developing countries are allocated quotas equivalent to their emission level from the previous period. The residual quotas, for developed countries, are allocated based on their share of emissions of the previous period. Note that in lines 10-12 where the quotas are set for the regions in the high-income coalition the quotas for the developing regions is subtracted from the overall cap using the lagged value of emissions and not the variable `emiQuota`. This is because the order of the loop over `ra` is undetermined and it is possible the the quotas for the developed regions will be set before the quotas for the developing regions.

$$EmiQY_{r,em} = \tau_{r,em}^{Emi} (EmiQ_{r,em} - EmiTot_{r,em}) \quad (E-11)$$

Listing 3.2: Example of a cap and trade regime

---

```

1  ifEmiCap      = 1 ;
2  emFlag("co2") = 1 ;
3  rq("wld")    = yes ;
4  emiCap.fx("wld", "co2", tsim) = 0.8*sum(r$mapr("wld", r), emiTot.l(r, "co2", tsim-1)*emiTot0(r, "co2"
   )) ;
5  ifEmiQuota(r) = 1 ;
6  loop(ra,
7     if(sameas(ra, "lmy"),
8        emiQuota.fx(r, "co2", tsim)$mapr(ra, r) = emiTot.l(r, "co2", tsim-1)*emiTot0(r, "co2") ;
9     elseif(sameas(ra, "hic"),
10        emiQuota.fx(r, "co2", tsim)$mapr(ra, r)
11        = (emiCap.l("wld", "co2", tsim)
12          - sum(rp$mapr("lmy", rp), emiTot.l(rp, "co2", tsim-1)*emiTot0(rp, "co2")))
13          * emiTot.l(r, "co2", tsim-1)*emiTot0(r, "co2")
14          / sum(rp$mapr(ra, rp), emiTot.l(rp, "co2", tsim-1)*emiTot0(rp, "co2")) ;
15     ) ;
16 ) ;
17 emiRegTax.lo("wld", "co2", tsim) = -inf ;
18 emiRegTax.up("wld", "co2", tsim) = +inf ;
19 emiTax.lo(r, "co2", tsim) = -inf ;
20 emiTax.up(r, "co2", tsim) = +inf ;
21 emiQuotaY.lo(r, "co2", tsim) = -inf ;
22 emiQuotaY.up(r, "co2", tsim) = +inf ;

```

---



## Chapter 4

# Model dynamics

The model as defined above represents the fully static version.<sup>1</sup> The factor stocks and technology are largely exogenous. This section describes the additional equations needed for the standard dynamics.

### 4.1 Labor supply

Labor is assumed to grow at some 'natural' rate for each skill level and by zone (in the case of segmented labor markets). At the moment, the natural rate of growth is equated to some measure of the growth in the relevant population and is assumed to be uniform for all skills and zones. The relevant population growth is defined by the following expression:

$$TLAB_{r,t} = \frac{\sum_p \phi_{r,p,t}^l Pop_{r,p,t}^t}{\sum_p \phi_{r,p,t-n}^l Pop_{r,p,t-n}^t} TLAB_{r,t-n}$$

where  $\phi^l$  is the relevant labor force participation rate. Population growth by cohort is assumed to be exogenous and is typically taken from a population scenario such as the UN or from those developed by the IAMC community known as the shared socio-economic pathways, or SSPs.

In the case of labor market segmentation with rural to urban migration, the level of migration is defined by equation (G-1). This simple migration equation is similar in spirit to the so-called Harris-Todaro model of migration.<sup>2</sup> Migration is a function of the expected urban wage relative to the expected rural wage, where  $\omega^m$  represents the migration elasticity.<sup>3</sup> Labor supply in each zone is equal to some exogenous component based on the 'natural' growth rate adjusted by migration, equation (G-2). The growth rate  $g^{lz}$  is typically exogenous and for the moment uniform across skills and zones. The parameter  $\delta^m$  is equal to -1 in the case of the rural zone and is equal to 1 for the urban zone. The parameter  $\mu^m$  is a multiplier for migration that accounts for migration occurring between periods in a multi-year time step. The level of the multiplier is given in equation (G-3). It is equal to 1 for single-year time steps. It is approximately equal to  $n$  for time steps of  $n$  years.

<sup>1</sup> In the model implementation, all of the equations listed till now are part of the **core** model—save for those that are substituted out.

<sup>2</sup> See [Harris and Todaro \(1970\)](#).

<sup>3</sup> There are more complex specifications of the migration function. See for example [Burniaux \(1987\)](#) and [Burniaux and van der Mensbrugge \(1991\)](#).

Equation (G-4) calculates implicitly the growth rate of labor by skill at the economy-wide level.

$$MIGR_{r,l} = \chi_{r,l}^m \left( \pi_{r,l}^{Urb} \right)^{\omega_{r,l}^m} \quad (G-1)$$

$$L_{r,l,z,t}^s = \left( 1 + g_{r,l,z,t}^{lz} \right)^n L_{r,l,z,t-n}^s + \delta^m \mu_{r,l,z,t}^m MIGR_{r,l,t} \quad (G-2)$$

$$\mu_{r,l,z,t}^m = \frac{\left( 1 + g_{r,l,z,t}^{lz} \right)^n \left( \frac{\pi_{r,l,t-n}^{Urb}}{\pi_{r,l,t}^{Urb}} \right)^{\omega_{r,l}^m} - 1}{\left( 1 + g_{r,l,z,t}^{lz} \right)^n \left( \frac{\pi_{r,l,t-n}^{Urb}}{\pi_{r,l,t}^{Urb}} \right)^{\omega_{r,l}^m/n} - 1} \quad (G-3)$$

$$LAB_{r,l,t}^s = \left( 1 + g_{r,l,t}^l \right)^n LAB_{r,l,t-n}^s \quad (G-4)$$

## 4.2 Capital supply

The total capital stock evolves according to the standard capital accumulation formula:

$$K_t^s = (1 - \delta_t) K_{t-1}^s + I_{t-1}$$

Since both terms on the right-hand side are lagged, with a single year time step the supply of capital is in effect exogenous. When moving to flexible time steps, cumulative investment over intermediate years needs to be accounted for. One simply way to do this is to assume a constant growth rate for investment between a past year and the current year:

$$I_t = (1 + g_t^I)^n I_{t-n}$$

After some algebra, this leads to a motion equation for the capital stock that takes the following form:<sup>4</sup>

$$K_t^s = (1 - \delta_t)^n K_{t-n}^s + \frac{(1 + g_t^I)^n - (1 - \delta_t)^n}{g_t^I + \delta_t} I_{t-n}$$

The resulting motion equation is no longer pre-determined as the contemporaneous level of investment is needed to measure the annual growth in investment. The equation is also quite sensitive to the relative values of  $g^I$  and  $\delta$ . In the model implementation, a somewhat transformed version of the motion equation is used that includes an additional equation that captures investment growth. Equation (G-5) is the investment growth factor. The value of the expression is equal to the inverse of the annual rate of growth of investment plus the depreciation rate. For example, when both are at 5 percent,  $\Psi$  takes the value 10. Equation (G-6) is the capital stock accumulation function for multi-period time steps. Both formulas depend on the contemporaneous level of investment. Equation (G-7) defines the normalized capital stock. It is a fixed scalar of the non-normalized level where the scaling factor is initialized using base year data, i.e.  $\chi_r^k = TK_{r,0}^s / K_{r,0}^s$

$$\Psi_{r,t} = \left[ \left( \frac{XFD_{r,inv,t}}{XFD_{r,inv,t-n}} \right)^{1/n} - 1 + \delta_{r,t} \right]^{-1} \quad (G-5)$$

$$K_{r,t}^s = [K_{r,t-n}^s - \Psi_{r,t} XFD_{r,inv,t-n}] (1 - \delta_{r,t})^n + \Psi_{r,t} XFD_{r,inv,t} \quad (G-6)$$

<sup>4</sup> It is easy to see that the equation collapses to the one-period motion equation when  $n$  is equal to 1.

$$TK_{r,t}^s = \chi_r^k K_{r,t}^s \quad (\text{G-7})$$

### 4.3 Technology shifts

By default, changes in technology are largely exogenous, for example autonomous energy efficiency improvement (AEEI) and improvements in international trade and transport services. The one exception is labor productivity. Equation (G-8) describes the growth in labor productivity. In the base year,  $\lambda^l$  is initialized to 1, and is set to grow at a rate of  $\gamma^l$  each year. If  $\alpha^{gl}$  is set to 0 and  $\beta^{gl}$  is set to 1, then labor productivity grows at a uniform rate across all activities. There are multiple strategies to put values on these parameters. These may include setting  $\beta^{gl}$  to 0 for a subset of activities and initializing  $\alpha^{gl}$  to some exogenous level based on external studies—for example setting  $\alpha^{gl}$  to 2 percent in the agricultural sectors. The remaining sectors could be segmented into two subsets—one with high productivity, e.g. set  $\alpha^{gl}$  to 2 percent, and one with low productivity, e.g. set  $\alpha^{gl}$  to 0, with  $\beta^{gl}$  set to 1 in both cases. In the baseline or reference scenario,  $\gamma^l$  will be calibrated to target a given level of per capita GDP growth and subject to the assumptions made on  $\alpha^{gl}$  and  $\beta^{gl}$ . In subsequent scenarios,  $\lambda^l$  will be exogenous having been calibrated in the reference scenario.

$$\lambda_{r,l,a,t}^l = \left(1 + \alpha_{r,l,a,t}^{gl} + \beta_{r,l,a,t}^{gl} \gamma_{r,t}^l\right)^n \lambda_{r,l,a,t-n}^l \quad (\text{G-8})$$

### 4.4 Preference shifters

#### 4.4.1 Armington twists

In the standard version of the model, the Armington preference parameters—both top and second levels—are held fixed at their base year calibrated levels. Thus changes in import shares are linked only to changes in relative prices. Historically, we have observed large changes in import shares without a concomitant change in relative prices. Dixon and Rimmer (2002) have developed a specification for the MONASH (and USAGE) model that ‘twists’ the relevant share parameters. The ‘twist’ targets a percentage change in the ratio of imports to domestic demand, which at unchanged component prices, is cost-neutral. The mechanics of the twist methodology is described in Appendix A.

For the ENVISAGE model, we have incorporated three versions of the twist—for the dual nested sourcing decision. The first two deal with the top-level Armington decision—purchases of domestic goods and the aggregate import bundle. Since we have two possible specifications—agent-based sourcing or national sourcing, we provide two twist specifications. The third is for the second level nest that sources aggregate imports by region of origin.

#### National sourcing

The equations for national sourcing of domestic purchases and aggregate imports are provided in equations (T-2) and (T-3), with the domestic and import share parameters given respectively by  $\alpha^{dt}$  and  $\alpha^{mt}$ . Equation (G-9) provides the ‘twisted’ domestic and imported share parameters where  $tw^{tl}$  is the level of the desired twist, i.e. the percent change in the ratio of imports to domestic purchases at unchanged prices. The twist has no impact on the share parameters if it is set to its default value of 0. The twist updating uses the (lagged) value share of imports in the top-level

Armington bundle.<sup>5</sup> Note that the formulas apply the twist factor directly to the share coefficients themselves and we drop the  $\lambda$  factor, i.e. preference change variable, from the formulas as these can be substituted out (refer to Appendix A).

$$\begin{aligned}\alpha_{r,i,t}^{dt} &= \alpha_{r,i,t-1}^{dt} \frac{1}{1 + s_{r,i,t-1}^{tI} tw_{r,i,t}^{tI}} \\ \alpha_{r,i,t}^{mt} &= \alpha_{r,i,t-1}^{mt} \frac{1 + tw_{r,i,t}^{tI}}{1 + s_{r,i,t-1}^{tI} tw_{r,i,t}^{tI}}\end{aligned}\tag{G-9}$$

where

$$s_{r,i,t-1}^{tI} = \frac{PMT_{r,i,t-1} XMT_{r,i,t-1}}{PAT_{r,i,t-1} XAT_{r,i,t-1}}$$

### Agent sourcing

The expressions for agent-based sourcing of domestic goods and aggregate imports are nearly identical with the inclusion of the index for the Armington agent,  $aa$ . The share parameters are part of equations (T-6) and (T-7). Equation (G-10) provides the relevant formulas for the 'twisted' shares using the appropriate import share level. Note that the twists are allowed to be agent-specific, though in practice, there is probably little evidence to differentiate across agents.

$$\begin{aligned}\alpha_{r,i,aa,t}^d &= \alpha_{r,i,aa,t-1}^d \frac{1}{1 + s_{r,i,aa,t-1}^I tw_{r,i,aa,t}^I} \\ \alpha_{r,i,aa,t}^m &= \alpha_{r,i,aa,t-1}^m \frac{1 + tw_{r,i,aa,t}^I}{1 + s_{r,i,aa,t-1}^I tw_{r,i,aa,t}^I}\end{aligned}\tag{G-10}$$

where

$$s_{r,i,aa,t-1}^I = \frac{PM_{r,i,aa,t-1} XM_{r,i,aa,t-1}}{PA_{r,i,aa,t-1} XA_{r,i,aa,t-1}}$$

### Second level Armington twists

The second level of the Armington nest allocates aggregate imports across source regions (see equation (T-13)). With the sole exception of a 2-region model, the standard twist does not apply and we thus apply the twist to a target set of source countries, i.e. we segment the source regions into 2.<sup>6</sup> The targeted regions will see their share increase (by the same twist), and the non-targeted regions will see their share decline.

$$\begin{aligned}\alpha_{s,i,r,t}^w &= \alpha_{s,i,r,t-1}^w \frac{1}{1 + s_{i,r,t-1}^2 tw_{r,i,t}^2} \quad \text{if } s \notin rtwtgt(s,r) \\ \alpha_{s,i,r,t}^w &= \alpha_{s,i,r,t-1}^w \frac{1 + tw_{r,i,t}^2}{1 + s_{i,r,t-1}^2 tw_{r,i,t}^2} \quad \text{if } s \in rtwtgt(s,r)\end{aligned}\tag{G-11}$$

where

$$s_{i,r,t-1}^2 = \frac{\sum_{s \in rtwtgt(s,r)} PDM_{s,i,r,t-1} XW_{s,i,r,t-1}}{\sum_s PDM_{s,i,r,t-1} XW_{s,i,r,t-1}}$$

<sup>5</sup> In the MONASH and USAGE models, the twist parameters are calculated contemporaneously, i.e. they are part of the model equations. In ENVISAGE, the twists are applied between periods. This should have only minimal impact on model results.

<sup>6</sup> The targeted regions can include only a single region.

## 4.5 Implementation

The `core` model includes all of the equations from Chapter 3.<sup>7</sup> There are two versions of the dynamic model. The reference simulation, that targets GDP growth, is defined as the `core` model plus the GDP growth equation (M-7), migration (G-1), labor supply by skill and zone (G-2), total labor supply by skill (G-4), investment growth factor (G-5), non-normalized capital stock (G-6), normalized capital stock (G-7), and labor productivity (G-8). This model is called `coreBaU` and it essentially calibrates the economy-wide labor productivity factor. All other dynamic simulations assume that labor productivity is exogenous i.e. per capita GDP growth is endogenous. This model is called `coreDyn` and includes the `core` model plus the GDP growth equation (M-7), migration (G-1), labor supply by skill and zone (G-2), total labor supply by skill (G-4), investment growth factor (G-5), non-normalized capital stock (G-6), and normalized capital stock (G-7).

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<sup>7</sup> With the exception of the GDP per capita growth equation as this requires a different MCP formulation.

# Appendix A

## The CES and CET functions

This appendix describes in full detail the two functional forms most widely used in CGE models—the constant-elasticity-of-substitution (CES) and constant-elasticity-of-transformation (CET) functions.

CES functions are widely used in demand functions where substitutability across different products and/or factors is needed and where the main objective is to minimize cost. CET functions are broadly used to determine supply functions across different markets where the main objective is to maximize revenues. The two are very similar in many ways and the algebraic derivations below will be more detailed for the CES function.

### A.1 The CES function

#### A.1.1 Basic formulas

In production, the CES function is used to select an optimal combination of inputs (either goods and/or factors) subject to a CES production function. In consumer demand, the CES is used as a utility (or sub-utility) or preference function. In either case, the purpose is to minimize the cost of purchasing the 'inputs' subject to the production or utility function. In generic terms the system takes the following form:

$$\min_{X_i} \sum_i P_i X_i$$

subject to the constraint:

$$V = A \left[ \sum_i a_i (\lambda_i X_i)^\rho \right]^{1/\rho}$$

The objective function represents aggregate expenditure. The constraint expression will be referred to as the CES primal function. The parameter  $A$  is an aggregate shifter that can be used to shift the overall production function (or utility function). Each input,  $X_i$ , is multiplied by an input-specific shifter,  $\lambda_i$ , that can be used to implement input-specific productivity increases (for example biased technological change), or specific changes in consumer preferences. The (primal) share coefficients,  $a_i$ , are typically calibrated to some base year data and held fixed. The CES exponent,  $\rho$ , is linked to the curvature of the CES function (and will be explained further below). For given component prices,  $P_i$ , and a given level of production or utility  $V$ , solving the optimization program above will yield optimal demand functions for the inputs,  $X_i$ .

The Lagrangian can be set up as:

$$\mathcal{L} = \sum_i P_i X_i + \Lambda \left( V - A \left[ \sum_i a_i (\lambda_i X_i)^\rho \right]^{1/\rho} \right)$$

Taking the partial derivative with respect to  $X_i$  and the Lagrange multiplier  $\Lambda$  yields the following system of equations:

$$P_i = \Lambda a_i \lambda_i^\rho X_i^{\rho-1} A \left[ \sum_i a_i (\lambda_i X_i)^\rho \right]^{(1-\rho)/\rho} = \Lambda a_i A^\rho \lambda_i^\rho X_i^{\rho-1} V^{1-\rho}$$

$$V = A \left[ \sum_i a_i (\Lambda_i X_i)^\rho \right]^{1/\rho}$$

Taking the first expression, it can be multiplied by  $X_i$ , and then summed. This of course is equal to the value of the bundle, i.e.  $P.V$ , where  $P$  is the aggregate price:

$$P.V = \sum_i P_i X_i = \Lambda V^{1-\rho} A^\rho \sum_i a_i \lambda_i^\rho X_i^\rho = \Lambda V^{1-\rho} V^\rho = \Lambda V$$

This shows that  $\Lambda$ , the Lagrange multiplier is the same as the aggregate price,  $P$ . We can re-arrange expression above to get an expression for optimal input demand, where  $\Lambda$  is replaced by  $P$ :

$$X_i = a_i^{1/(1-\rho)} A^{\rho/(1-\rho)} \left( \frac{P}{P_i} \right)^{1/(1-\rho)} \lambda_i^{\rho/(1-\rho)} V$$

We finally end up with the following expression, where the CES primal exponent,  $\rho$ , is replaced by the so-called CES elasticity of substitution,  $\sigma$ :

$$X_i = \alpha_i (A \lambda_i)^{\sigma-1} \left( \frac{P}{P_i} \right)^\sigma V \quad (\text{A-1})$$

where we made the following substitutions:

$$\sigma = \frac{1}{1-\rho} \Leftrightarrow \rho = \frac{\sigma-1}{\sigma} \Leftrightarrow \frac{\rho}{1-\rho} = \sigma-1 \Leftrightarrow \rho \cdot \sigma = \sigma-1$$

and

$$\alpha_i = a_i^{1/(1-\rho)} = a_i^\sigma \Leftrightarrow a_i = \alpha_i^{1/\sigma}$$

Abstracting from the technology parameters, the demand equation implies that demand for 'input'  $X_i$  is a (volume) share of total demand  $V$ . The share, with equal prices is simply equal to  $\alpha_i$ . With a positive elasticity of substitution, the share is sensitive to the ratio of prices relative to the aggregate price index. Since the component price is in the denominator, the demand for that component declines if its price rises relative to the average and vice versa if its price declines vis-à-vis the average price. The  $\alpha$  parameters will be referred to as the CES dual share parameters (for reasons described below), and the  $a$  parameters are the primal CES share parameters. Notice that expression (A-1) simplifies if it is expressed in terms of efficiency inputs,  $X^e$  and efficiency prices,  $P^e$ :

$$X_i^e = \alpha_i A^{\sigma-1} \left( \frac{P}{P_i^e} \right)^\sigma V$$

where

$$X_i^e = \lambda_i X_i$$

and

$$P_i^e = \frac{P_i}{\lambda_i}$$

The aggregate price  $P$  can be determined using two expressions. The first is the zero profit condition:

$$P = \frac{\sum_i P_i X_i}{V}$$

The other is by inserting the optimal demand relation  $X_i$  (equation A-1) in the zero profit condition :

$$P.V = \sum_i P_i X_i = A^{\sigma-1} \sum_i P_i \alpha_i \left( \frac{P}{P_i} \right)^\sigma \lambda_i^{\sigma-1} V = P^\sigma A^{\sigma-1} V \sum_i \alpha_i \left( \frac{P_i}{\lambda_i} \right)^{1-\sigma}$$

The  $V$ 's cancel out, and the aggregate price can then be expressed by the following formula:

$$P = \frac{1}{A} \left[ \sum_i \alpha_i \left( \frac{P_i}{\lambda_i} \right)^{1-\sigma} \right]^{1/(1-\sigma)} = \frac{1}{A} \left[ \sum_i \alpha_i (P_i^e)^{1-\sigma} \right]^{1/(1-\sigma)} \quad (\text{A-2})$$

This is sometimes referred to as the dual price expression. It has virtually the same functional form as the CES primal, which is a CES aggregation of the input volumes using the primal share parameters as weights. The CES dual price formula is a CES aggregation of the input prices using the CES dual share parameters as weights and a different exponent. In a CGE model, the zero-profit condition or the dual price formula can be used interchangeably (with the proviso that the substitution elasticity differs from 1).<sup>1</sup> There is a simple formula for the budget shares given by:

$$s_i = \frac{P_i X_i}{P.V} = \alpha_i (A \lambda_i)^{\sigma-1} \left( \frac{P}{P_i} \right)^\sigma V \left( \frac{P_i}{P} \right) \frac{1}{V} = \alpha_i (A \lambda_i)^{\sigma-1} \left( \frac{P}{P_i} \right)^{\sigma-1} \quad (\text{A-3})$$

Notice that this expression for the budget shares is only a function of prices. With the technology parameters set to 1, this simplifies further to:

$$s_i = \alpha_i \left( \frac{P}{P_i} \right)^{\sigma-1}$$

It turns out that the parameter  $\sigma$  measures the elasticity of substitution for the CES function and is constant over the entire domain. The elasticity of substitution is an indication of the curvature of an isoquant, see [Varian \(1992\)](#), i.e. it measures the rate of change of the ratio of inputs (in a 2-input case), relative to the change in their relative prices. For example, if the CES

<sup>1</sup> We shall see below that when the substitution elasticity is 1, both primal and dual expressions take a different functional form.



combines capital and labor to form output, a large substitution elasticity suggests that the factor proportions will change rapidly as one of the inputs becomes cheaper relative to the other. There are two limiting cases of interest. If the substitution elasticity is zero, then there is no substitution across inputs and the optimal choice is to use them in fixed proportion. At the other extreme, if the substitution elasticity is infinite, this is equivalent to saying the inputs are identical, and in this case, in equilibrium, the two inputs would have the same price. This could potentially be the case for electricity production. If there is a regional or national buyer of electricity, the buyer is most likely indifferent about how the electricity is produced and thus will purchase from the lowest cost producer (a perhaps somewhat simplified view of electricity markets.) This implies that the cost of the electricity inputs, from all sources (e.g. thermal, nuclear, etc.) would be (nearly) identical.

The elasticity of substitution across inputs is defined by the following formula:

$$\sigma = \frac{\partial \left( \frac{X_i}{X_j} \right) \left( \frac{P_i}{P_j} \right)}{\partial \left( \frac{P_i}{P_j} \right) \left( \frac{X_i}{X_j} \right)}$$

The ratio of the optimal inputs using expression (A-1) is:

$$\frac{\alpha_i}{\alpha_j} \left( \frac{P_i}{P_j} \right)^{-\sigma} \left( \frac{\lambda_i}{\lambda_j} \right)^{\sigma-1}$$

Taking the partial derivative of the expression with respect to the ratio  $P_i/P_j$  and multiplying it by the second term of the elasticity of substitution yields the conclusion that the substitution elasticity is  $-\sigma$ . It is logical that it is negative. If the price of one input increases, say labor, relative to the other, say capital, producers would substitute away from labor towards capital, i.e. the ratio of labor to capital would drop as the price of labor increases relative to capital. [Varian \(1992\)](#) in fact defines the elasticity of substitution in terms of the absolute value of the technical rate of substitution, that measures the slope of the budget line. Numerically what it represents is the relative change in the ratios. If  $\sigma$  is 1, for example, and the price of labor increases by 10 percent relative to capital, the labor to capital ratio would decrease by (around) 10 percent.<sup>2</sup> The higher is  $\sigma$ , the more the proportion changes.

### A.1.2 Special cases

There are three special cases that require additional derivations due to numerical restrictions on the primal and dual exponents. A substitution elasticity of 0 is clearly a special case and is referred to as a Leontief technology. From the dual price formula, it is clear that  $\sigma$  equal to 1 is a special case and is known as a Cobb-Douglas technology (or utility function). Finally, a value of  $\rho$  equal to 1 corresponds to infinite substitution elasticity and a linear primal aggregation function. This is also referred to as a case of perfect substitution.

#### The Leontief case

The first special case is for the so-called Leontief functional form.<sup>3</sup> In this case the substitution elasticity is 0 and corresponds to a value for  $\rho$  that is  $-\infty$ . In this case the optimization program takes the following form:<sup>4</sup>

<sup>2</sup> The elasticity is a marginal concept that holds only approximately for large changes.

<sup>3</sup> Leontief, winner of the 1973 Nobel prize in Economics, is renowned for his work on input-output tables, much of which focused on fixed input technologies (!!!! reference).

<sup>4</sup> !!!! need a reference

$$\min_{X_i} \sum_i P_i X_i$$

subject to the constraint:

$$V = \min \left( \frac{\alpha_i}{\lambda_i X_i} \right)$$

The visual implementation has L-shaped isoquants. The Leontief technology constraint or production/utility function is discontinuous. Fortunately, the optimal demand functions are easy to implement and are just special cases of expression (A-1):

$$X_i = \frac{\alpha_i V}{\lambda_i A}$$

$$P = \frac{1}{A} \sum_i \alpha_i \left( \frac{P_i}{\lambda_i} \right)$$

Thus the Leontief specification implies that inputs are always in fixed proportion relative to output and the aggregate price is simply the linear weighted aggregation of the input prices, where the weights are given by the input-output coefficients, adjusted by changes in efficiency. The efficiency parameter has a nice intuitive interpretation in this case. Say  $\lambda$  increases by 10 percent, then demand for the input declines by 10 percent.

### The Cobb-Douglas function

Another special case is the so-called Cobb-Douglas function, very frequently used in introductory text books in microeconomics. The Cobb-Douglas function has a substitution elasticity of 1 implying that  $\rho$  is equal to 0. Clearly, this creates a problem for specifying the CES primal function as well as the CES dual price function. As with the Leontief, the optimal demand conditions are given by expression (A-1), with  $\sigma$  set to 1:

$$X_i = \alpha_i \left( \frac{P}{P_i} \right) V \Leftrightarrow s_i = \frac{P_i X_i}{P \cdot V} = \alpha_i$$

The Cobb-Douglas specification has constant budget shares irrespective of relative prices (and changes in technology). Another implication of the Cobb-Douglas specification is that the dual shares must add up to 1 as they are equivalent to the budget shares. By definition, as well, the primal and dual shares are the same. The Cobb-Douglas primal and dual price functions have the following expressions:

$$V = A \prod_i (\lambda_i X_i)^{\alpha_i}$$

$$P = \frac{1}{A} \prod_i \left( \frac{P_i}{\alpha_i \lambda_i} \right)^{\alpha_i}$$

Rather than code the Cobb-Douglas function as a special case, many modelers choose to replace the elasticity of 1 with a value close to 1 such as 1.01. This would have only marginal repercussions on the results.

### Perfect substitution

The third special case is for a substitution elasticity of infinity. In this case  $\rho$  takes the value of 1 and the primal function is a straight linear aggregation of the inputs. The optimal demand conditions cannot be used in the case of an infinite substitution elasticity. In its stead, the optimal demand condition is replaced with the law-of-one-price, adjusted by efficiency differentials, and the zero profit condition is replaced with the CES primal function, i.e. the linear weighted aggregation of the inputs:

$$\frac{P_i}{\alpha_i \lambda_i} = P$$

$$V = \sum_i \alpha_i \lambda_i X_i$$

The aggregation function can be replaced by the zero profit condition:<sup>5</sup>

$$P.V = \sum_i P_i X_i$$

#### A.1.3 Calibration of the CES function

Calibration typically involves inverting functional forms to evaluate the value of a parameter given initial values for variables. Prices and volumes,  $P_i$ ,  $X_i$ ,  $V$  and  $P$ , are normally initialized to a given database or SAM. This may or may not include actual price/volume splits. If not, prices will typically be initialized at unit value—potentially adjusted for a price wedge such as a tax or a margin. The substitution elasticities are also normally inputs—either derived from econometric estimation, other data bases or models, or from a literature review. This leaves the parameters  $\lambda_i$ ,  $\alpha_i$  and  $A$  to calibrate. The technology parameters are normally associated with dynamics, so there is little reason not to initialize them to unit value as they can be incorporated in the initial share parameter value without any loss in generality. Thus, the only parameters left to calibrate are the  $\alpha_i$  from which it is possible to derive the primal share parameters,  $a_i$ , if needed. The calibration formula is derived from the inversion of equation (A-1):

$$\alpha_i = \left( \frac{X_i}{V} \right) \left( \frac{P_i}{P} \right)^\sigma (A \cdot \lambda_i)^{1-\sigma} = \left( \frac{X_i}{V} \right) \left( \frac{P_i}{P} \right)^\sigma$$

The right-most term is the most used formula where the technology parameters are explicitly set to 1.<sup>6</sup>

<sup>5</sup> Modelers have the choice of using the primal aggregation function or the revenue function. The latter holds in all three special cases for the substitution elasticity.

<sup>6</sup> In many introductions to CGE models, the calibration formulas explicitly exclude the price term. This is a dangerous practice that can lead to model bugs that can be hard to detect. It is best to explicitly initialize prices to 1 and use the correct calibration formula. In fact, one way to test model calibration and specification is to initialize prices to an arbitrary value and initialize volumes subject to these prices. Simulating a counterfactual with no shocks should replicate the initial data solution. If not, there is an error in initialization, calibration and/or specification.

### A.1.4 Comparative statics

#### Elasticities

This section will derive some of the key elasticities of the CES function. The first relationship is the elasticity of the aggregate price with respect to a component price:

$$\frac{\partial P}{\partial P_i} \frac{P_i}{P} = s_i = \frac{P_i X_i}{P.V}$$

The elasticity of the aggregate price relative to an input price is equal to the budget share, irrespective of the substitution elasticity. The matrix of own- and cross-price elasticities, holding the aggregate volume constant is given by the following formula:

$$\varepsilon_{ij} = \frac{\partial X_i}{\partial P_j} \frac{P_j}{X_i} = \sigma(s_j - \delta_{ij})$$

where  $\delta_{ij}$  is the so-called Kronecker's delta that takes the value 1 for  $i$  equal to  $j$ , else it takes the value 0. Since  $\sigma$  is positive, all components are gross substitutes in the CES.

#### Formulas in percent differences

It is useful in terms of comparative static analyses to convert the basic equations into percent differences. It is easy to trace out the impacts of a change in one of the 'exogenous' variables on demand and the overall price index. This is also the form of the equations used for models implemented in GEMPACK such as MONASH-style models.

The following expressions convey expressions (A-1) and (A-2) into their percent difference form:

$$\begin{aligned} \frac{\partial X_i}{X_i} = \dot{X}_i &= \dot{V} + \sigma \left( \dot{P} - \dot{P}_i \right) - (\sigma - 1) \left( \dot{A} + \dot{\lambda}_i \right) \\ \frac{\partial P}{P} = \dot{P} &= -\dot{A} + \sum_i s_i \dot{P}_i - \sum_i s_i \dot{\lambda}_i = -\dot{A} + \sum_i s_i \left( \dot{P}_i - \dot{\lambda}_i \right) \end{aligned}$$

Thus the percent change in the unit cost,  $P$ , for a change in the input price,  $P_i$ , all else equal, is (approximately) the value share of component  $i$ —as already noted above.

### A.1.5 Growth Accounting

Use can be made of the linearization above to derive the linearized growth accounting formula:

$$\frac{\Delta V}{V} = \frac{\Delta A}{A} + \sum_i s_i \frac{\Delta x_i}{x_i} + \sum_i s_i \frac{\Delta \lambda_i}{\lambda_i}$$

### A.1.6 Parameter twists

#### The basic analytics

This final section on the CES describes how to adjust the share parameters in a dynamic scenario under a specific assumption—this is called the twist adjustment and is a core feature of the dynamic MONASH model, see [Dixon and Rimmer \(2002\)](#). The basic idea is to alter the share parameter, in a two-component CES, to target a given change in the ratio of the two components, however, with

neutral impacts on the aggregate cost. For example, the target may be a cost-neutral increase in the capital/labor ratio by  $x\%$ , or an increase in the import to domestic ratio of  $y\%$ .

The ratio of the two components is given by the following expression using equation (A-1) as the starting point:

$$R = \frac{\alpha_1 \lambda_1^{\sigma-1} P_2^\sigma}{\alpha_2 \lambda_2^{\sigma-1} P_1^\sigma}$$

The idea is to move the initial ratio,  $R_{t-1}$  to  $R_t$  by  $tw$  percent.

$$\frac{R_t}{R_{t-1}} = (1 + tw_t)$$

Using the formulas above, we have:

$$\frac{R_t}{R_{t-1}} = (1 + tw_t) = \frac{\left(\frac{\lambda_{1,t}}{\lambda_{1,t-1}}\right)^{\sigma-1}}{\left(\frac{\lambda_{2,t}}{\lambda_{2,t-1}}\right)^{\sigma-1}} = \frac{(1 + \pi_{1,t})^{\sigma-1}}{(1 + \pi_{2,t})^{\sigma-1}}$$

The  $\pi$  variables represent the growth (either positive or negative) that will be applied to the technology parameters under the assumption of cost-neutral technological change. We can start with the dual cost function for year  $t$ , but with year  $t - 1$  prices:

$$\begin{aligned} P_{t-1}^{1-\sigma} &= \alpha_1 \left(\frac{P_{1,t-1}}{\lambda_{1,t}}\right)^{1-\sigma} + \alpha_2 \left(\frac{P_{2,t-1}}{\lambda_{2,t}}\right)^{1-\sigma} \\ &= \alpha_1 (1 + \pi_{1,t})^{\sigma-1} \left(\frac{P_{1,t-1}}{\lambda_{1,t}}\right)^{1-\sigma} + \alpha_2 (1 + \pi_{2,t})^{\sigma-1} \left(\frac{P_{2,t-1}}{\lambda_{2,t}}\right)^{1-\sigma} \end{aligned}$$

Recall that the share equation is given by:

$$s_{i,t-1} = \alpha_i \lambda_{i,t-1}^{\sigma-1} \left(\frac{P_t}{P_{i,t-1}}\right)^{\sigma-1}$$

Dividing through the expression above by  $P_t^{1-\sigma}$  and inserting the share expressions for year  $t - 1$ , we end up with:

$$1 = s_{1,t-1}(1 + \pi_{1,t})^{\sigma-1} + s_{2,t-1}(1 + \pi_{2,t})^{\sigma-1}$$

Solving in terms of  $\pi_1$ , we have:

$$(1 + \pi_{1,t})^{\sigma-1} = \frac{1 - s_{2,t-1}(1 + \pi_{2,t})^{\sigma-1}}{s_{1,t-1}}$$

and this can be inserted into the twist target formula to get:

$$1 + tw_t = \frac{1 - s_{2,t-1}(1 + \pi_{2,t})^{\sigma-1}}{s_{1,t-1}(1 + \pi_{2,t})^{\sigma-1}} = \frac{(1 + \pi_{2,t})^{1-\sigma} - s_{2,t-1}}{s_{1,t-1}}$$

Finally,  $\pi_2$  can be isolated to yield:

$$1 + \pi_{2,t} = [s_{1,t-1}(1 + tw_t) + s_{2,t-1}]^{1/(1-\sigma)} = [1 + s_{1,t-1}tw_t]^{1/(1-\sigma)}$$

We can re-insert this into the expression above to derive an expression for  $\pi_1$ :

$$1 + \pi_{1,t} = \left[ \frac{1 + s_{1,t-1}tw_t}{1 + tw_t} \right]^{1/(1-\sigma)}$$

Finally, the productivity update formulas that incorporate the twist adjustment take the form:

$$\lambda_{1,t} = (1 + \pi_{1,t})\lambda_{1,t-1} = \left[ \frac{1 + s_{1,t-1}tw_t}{1 + tw_t} \right]^{1/(1-\sigma)} \lambda_{1,t-1}$$

$$\lambda_{2,t} = (1 + \pi_{2,t})\lambda_{2,t-1} = [1 + s_{1,t-1}tw_t]^{1/(1-\sigma)} \lambda_{2,t-1}$$

It is possible to generalize these formulas by partitioning the set of CES components into two sets—a set indexed by 1 that is the target set, and a set indexed by 2 that is the complement. For example, think of a set of electricity technologies that includes conventional and advanced. It is possible then to provide the same twist to all of the new technologies relative to the conventional technologies. The only change in the formulas above is that the share variable for the single component is replaced by the sum of the shares for the bundle of components:

$$\lambda_{1,t} = (1 + \pi_{1,t})\lambda_{1,t-1} = \left[ \frac{1 + tw_t \sum_{i \in 1} s_{i,t-1}}{1 + tw_t} \right]^{1/(1-\sigma)} \lambda_{1,t-1}$$

$$\lambda_{2,t} = (1 + \pi_{2,t})\lambda_{2,t-1} = \left[ 1 + tw_t \sum_{i \in 1} s_{i,t-1} \right]^{1/(1-\sigma)} \lambda_{2,t-1}$$

### Converting to percent differences

The  $\pi$  factors reflect a percentage change in the relevant productivity factors for each of the components. Using a Taylor series approximation, the formulas above can be converted to a linear equation that is used by the Monash-style models. For the first component, we have:

$$\pi_1 = F(tw) = \left[ \frac{1 + s_1tw}{1 + tw} \right]^{1/(1-\sigma)} - 1 \approx F(0) + tw.F'(0) = -tw \frac{1 - s_1}{1 - \sigma}$$

For the second component we have:

$$\pi_2 = F(tw) = [1 + s_1tw]^{1/(1-\sigma)} - 1 \approx F(0) + tw.F'(0) = tw \frac{s_1}{1 - \sigma}$$

Note that in the Monash models, the signs are reversed because the productivity factors divide the volume components whereas in the formulation above the productivity factors are multiplicative.

### Examples of twisting the share parameters

We demonstrate these concepts with two examples. The first is a CES production function of capital and labor, where the labor share is 60% and the capital/labor substitution elasticity (i.e.  $\sigma$ ) is set to 0.9. Prices are initialized at 1, therefore the original capital/labor ratio is 2/3. The target is to raise the capital/labor ratio 10% assuming cost neutrality. Table A.1 shows the key results. Labor efficiency would increase by 48% and capital efficiency would decline by 43%.

The second example comes from trade and the Armington assumption. Assume an 80/20 split between domestic goods and imports in value and volume implying a ratio of imports to demand of domestic goods of 0.25. Table A.2 shows the twist parameters needed to achieve an increase in this ratio of 10 percent with an Armington elasticity of 2. The preference parameter for imports increases by nearly 8 percent, while that for domestic goods decreases by 2 percent.

Table A.1: Example of capital/labor twist

	Labor	Capital	Capital/labor ratio
Initial	60.0	40.0	0.6667
After twist	57.7	42.3	0.7333
Percent change	-3.8	5.8	10.0
Growth factor	0.48	-0.43	

Table A.2: Example of Armington import/domestic twist

	Domestic	Import	Import/domestic ratio
Initial	80.0	20.0	0.250
After twist	78.4	21.6	0.275
Percent change	-2.0	7.8	10.0
Growth factor	-0.02	0.08	

### A.1.7 Summary

In summary, the CES functional form is often used as a production (or sub-production) function that combines two or more inputs to form output (or an intermediate composite bundle), under the assumption of cost minimization. It is also frequently used to maximize utility (or sub-utility) over a set of two or more goods, again with the assumption of cost minimization. Table A.3 highlights the two main expressions to emerge from the optimization—the derived demand functions,  $X_i$ , and the CES dual price expression,  $P$ . The top row shows the expression with all technology parameters initialized at 1, and the bottom row the most generic version.

Table A.3: Key equations for CES implementation

	Demand	Aggregate price
Basic	$X_i = \alpha_i V \left( \frac{P}{P_i} \right)^\sigma$	$P = \left[ \sum_i \alpha_i P_i^{1-\sigma} \right]^{\frac{1}{1-\sigma}}$
with full technology	$X_i = \alpha_i (A \lambda_i)^{\sigma-1} V \left( \frac{P}{P_i} \right)^\sigma$	$P = \frac{1}{A} \left[ \sum_i \alpha_i \left( \frac{P}{\lambda_i} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}$

## A.2 The CET Function

### A.2.1 The basic formulation

This section describes the constant-elasticity-of-transformation (CET) function. The CET function is often used to describe a transformation frontier between two or more outputs. For example, a producer may produce two or more products and decides how much of each to produce based on market conditions, i.e. relative prices. The CET is often used to represent a producer’s decision on the allocation of output between domestic and foreign markets. Another example is land supply, where land will be allocated across different uses according to the relative returns. The transformation elasticity is assumed to be uniform between any pair of outputs and therefore is analogous to the demand-based CES function described in detail above. The exposition of the CET will be

much more succinct than that of the CES because most of the derivations can be derived in a similar fashion.

The CET can be setup as a revenue maximization problem, subject to a transformation frontier:

$$\max_{X_i} \sum_i P_i X_i$$

subject to

$$V = A \left[ \sum_i g_i (\lambda_i X_i)^v \right]^{1/v}$$

where  $V$  is the aggregate volume (e.g. aggregate supply),  $X_i$  are the relevant components (sector-specific supply),  $P_i$  are the corresponding prices,  $g_i$  are the CET (primal) share parameters, and  $\nu$  is the CET exponent. The CET exponent is related to the CET transformation elasticity,  $\omega$  via the following relation:

$$\nu = \frac{\omega + 1}{\omega} \Leftrightarrow \omega = \frac{1}{\nu - 1}$$

The transformation elasticity is assumed to be positive. Solution of this maximization problem leads to the following first order conditions:

$$X_i = \gamma_i (A \lambda_i)^{-1-\omega} \left( \frac{P_i}{P} \right)^\omega V \tag{A-4}$$

and

$$P = \frac{1}{A} \left[ \sum_i \gamma_i \left( \frac{P_i}{\lambda_i} \right)^{1+\omega} \right]^{1/(1+\omega)} \tag{A-5}$$

where the  $\gamma_i$  parameters are related to the primal share parameters,  $g_i$ , by the following formula:

$$\gamma_i = g_i^{-\omega} \Leftrightarrow g_i = \left( \frac{1}{\gamma_i} \right)^{1/\omega}$$

From expression A-4, and ignoring the technology parameters for the moment, the clear difference with the CES expression for optimal demand (equation A-1) is that the component price is in the numerator and the aggregate price in the denominator. This is intuitively logical. If the supply price to a market goes up relative to the average market price, one would anticipate that supply would increase to that market. The greater the transformation elasticity the greater are the market shifts.

Calibration is similar to the CES case. Prices and volumes are initialized using base year data. Equation (A-4) can then be inverted to calculate the share parameters,  $\gamma_i$ , with typically the technology parameters initialized to the value 1. In most implementations, there is no need to carry around the primal share parameters, nor the primal exponent.

The main interesting case for the CET is the case of perfect transformation, i.e. the transformation elasticity is infinity. In this case the CET exponent is 0 and the aggregation function is a linear weighted aggregation of the components. The standard CET equations are then replaced by the law-of-one price and the linear aggregation function (or alternatively, the zero profit condition).



$$\frac{P_i}{A\lambda_i} = P\forall i$$

$$A \sum_i \lambda_i X_i = X$$

### A.2.2 Converting to percent differences

It is easier to interpret or decompose the results of a simulation by looking at the CET equations in percent differences form—that is the standard form for MONASH-style models and implementation in GEMPACK. The following equations show the equations in percent difference form:

$$\frac{\partial X_i}{X_i} = \dot{X}_i = \dot{V} + \omega \left( \dot{P}_i - \dot{P} \right) - (\omega + 1) \left( \dot{\lambda}_i + \dot{A} \right)$$

$$\frac{\partial P}{P} = \dot{P} = -\dot{A} + \sum_i s_i \dot{P}_i - \sum_i s_i \dot{\lambda}_i = -\dot{A} + \sum_i s_i \left( \dot{P}_i - \dot{\lambda}_i \right)$$

where the variable  $s_i$  is the value share of component  $i$  in total revenue:

$$s_i = \frac{P_i X_i}{P \cdot V} = \gamma_i \left( \frac{P_i}{A \cdot \lambda_i \cdot P} \right)^{\omega+1}$$

## A.3 Modified CES and CET that incorporate additivity

The standard CET supply allocation specification does not preserve physical additivity, i.e. the sum of the volume components is not necessarily equal to the total volume. There are a number of alternative specifications that do preserve volume homogeneity, for example the multinomial logit. One alternative, described below, uses a modified form of the CET preference function. This specification has been used for labor and land supply allocations (see respectively [Dixon and Rimmer \(2006\)](#) and [Giesecke et al. \(2013\)](#)).

### A.3.1 The CET implementation

The CET alternative involves solving the following optimization:

$$\max_{X_i} U = \left[ \sum_i g_i (\lambda_i P_i X_i)^\nu \right]^{1/\nu}$$

subject to the constraint:

$$V = \sum_i X_i$$

The variable definitions are similar to above,  $X_i$  are the volume components,  $P_i$  are the relevant component prices and  $V$  is aggregate volume. The  $\lambda_i$  parameters are preference parameters. The CET utility function is not simply a function of the volumes, but explicitly a function of the

preference-adjusted revenues of the individual components. The closed-form solution to the above system is the following set of equations:

$$X_i = \gamma_i V \left( \frac{\lambda_i P_i}{P^c} \right)^\omega \quad (\text{A-6})$$

$$P^c = \left[ \sum_i \gamma_i (\lambda_i P_i)^\omega \right]^{1/\omega} \quad (\text{A-7})$$

Both equations are similar to their standard CET counterparts, but with some differences. First,  $P^c$  is a price index, but it is not the average price of the components, i.e.  $P^c X \neq \sum_i P_i X_i$ . Second, this price index is based on  $\omega$  not  $1 + \omega$  as in the standard CET dual price expression. The revenue correct price index is defined by the following formula:

$$P = \frac{\sum_i \gamma_i \lambda_i^\omega P_i^{\omega+1}}{\sum_i \gamma_i \lambda_i^\omega P_i^\omega} = \frac{\sum_i \gamma_i \lambda_i^\omega P_i^{\omega+1}}{(P^c)^\omega} = \sum_i \gamma_i P_i \left( \frac{\lambda_i P_i}{P^c} \right)^\omega = \sum_i \frac{X_i}{V} P_i \quad (\text{A-8})$$

The other transformations include:

$$\gamma_i = g_i^{1+\omega}$$

$$\omega = \frac{\nu}{1-\nu} \iff \nu = \frac{\omega}{1+\omega}$$

It is worth noting that the relation between  $\omega$  and  $\nu$  differs from the standard CET relation as the respective formula is inverted. The implication of this is that  $\nu$  is bounded below by 0 instead of  $\infty$ , but is otherwise positive over the entire (positive) range of  $\omega$ . And, in both the standard and adjusted CET  $\nu$  converges to 1 as  $\omega$  converges to  $\infty$ . As regards calibration, there is an extra degree of freedom as the value for utility is not specified. It is easiest to simply set  $P^c$  to 1 as for given  $P_i$  and  $\lambda_i$  the calibration of the  $\gamma$  parameters is straightforward:

$$\gamma_i = \frac{X_i}{V} \left( \frac{\lambda_i P_i}{P^c} \right)^{-\omega}$$

If prices and technology or preference parameters are initialized at 1, the calibrated  $\gamma$  parameters are equal to the initial volume shares.

Converting this to a Monash-style equation in percent differences, the derived supply function is:

$$\dot{X}_i = \dot{V} + \omega \left[ \dot{P}_i + \dot{\lambda}_i - \sum_{j=1}^n \frac{X_j}{V} (\dot{P}_j + \dot{\lambda}_j) \right]$$

This equation uses volume shares as weights for cross-price (and cross-preference) effects. In the standard CET formulation, value shares are used as weights.

The standard specification needs some modifications for two special cases—perfect transformation and perfect immobility. The case of perfect transformation, i.e. a transformation elasticity of  $\infty$ , leads to all prices moving in unison with the aggregate price index. Thus equation (A-6) is replaced with the following expression:

$$\lambda_i P_i = P^c$$

where  $\lambda_i$  is calibrated to the initial price ratios. The price index expression, equation (A-7) is replaced with the volume constraint:

$$V = \sum_i X_i$$

In the model implementation of the adjusted CET, this latter expression can be used in all cases and can replace equation (A-7).

The case of zero mobility is readily implemented by dropping completely equation (A-7) (or its equivalent, i.e. the volume adding up constraint). With a transformation elasticity of 0, the price composite index in equation (A-6) simply drops out and the volume components are in strict proportion to the aggregate volume.

### A.3.2 The CES implementation

The adjusted CET (and CRETH) functions replace their counterparts for the allocation problem that preserves additivity. Analogous specifications exist for the CES and CRESH functions that emulate the implementation of their standard counterparts but also allow for additivity.

The CES alternative involves solving the following optimization:

$$\min_{X_i} U = \left[ \sum_i a_i (\lambda_i P_i X_i)^\rho \right]^{1/\rho}$$

subject to the constraint:

$$V = \sum_i X_i$$

As in the case of the adjusted CET, the adjusted CES utility function is a function of the preference adjusted cost components. The closed-form solution to the above system is the following set of equations:

$$X_i = \alpha_i V \left( \frac{P^c}{\lambda_i P_i} \right)^\sigma \quad (\text{A-9})$$

$$P^c = \left[ \sum_i \alpha_i (\lambda_i P_i)^{-\sigma} \right]^{-1/\sigma} \quad (\text{A-10})$$

Both equations are similar to their standard CES counterparts, but with some differences. First,  $P^c$  is a price index, but it is not the average price of the components, i.e.  $P^c X \neq \sum_i P_i X_i$ . Second, this price index is based on  $-\sigma$  not  $1 - \sigma$  as in the standard CES dual price expression. The revenue correct price index is defined by the following formula:

$$P = \frac{\sum_i \alpha_i \lambda_i^\sigma P_i^{1-\sigma}}{\sum_i \alpha_i \lambda_i^\sigma P_i^\sigma} = \frac{\sum_i \alpha_i \lambda_i^\sigma P_i^{1-\sigma}}{(P^c)^{-\sigma}} = \sum_i \alpha_i P_i \left( \frac{\lambda_i P_i}{P^c} \right)^{-\sigma} = \sum_i \frac{X_i}{V} P_i \quad (\text{A-11})$$

The other transformations include:

$$\alpha_i = a_i^{1-\sigma}$$

$$\sigma = \frac{\rho}{\rho - 1} \iff \rho = \frac{\sigma}{\sigma - 1}$$

It is worth noting that the relation between  $\sigma$  and  $\rho$  differs from the standard CES relation as the respective formula is inverted. The implication of this is that  $\rho$  is bounded below by 0 instead of  $-\infty$ . It decreases towards  $-\infty$ , as  $\sigma$  increases towards 1, which is a discontinuity point. It decreases from  $\infty$  towards 1 as  $\sigma$  increases from 1 to  $\infty$ .

It is relatively easy to show that the following simple expression holds for the utility function:

$$U = P^c V \tag{A-12}$$

As regards calibration, there is an extra degree of freedom as the value for utility is not specified. It is easiest to simply set  $P^c$  to 1 as for given  $P_i$  and  $\lambda_i$  the calibration of the  $\alpha$  parameters is straightforward:

$$\alpha_i = \frac{X_i}{V} \left( \frac{\lambda_i P_i}{P^c} \right)^\sigma$$

If prices and technology or preference parameters are initialized at 1, the calibrated  $\alpha$  parameters are equal to the initial volume shares.

Converting this to a Monash-style equation in percent differences, the derived demand function is:

$$\dot{X}_i = \dot{V} - \sigma \left[ \dot{P}_i + \dot{\lambda}_i - \sum_{j=1}^n \frac{X_j}{V} (\dot{P}_j + \dot{\lambda}_j) \right]$$

This equation uses volume shares as weights for cross-price (and cross-preference) effects. In the standard CES formulation, value shares are used as weights.

### A.3.3 Using twists with the adjusted CES

The 'twist' idea described for the normal CES can be applied to the adjusted CES. The concept is somewhat different given the type of optimization problem posed. Rather than change the share parameters in a given direction with cost neutrality, the idea is to change the share parameters with utility neutrality. The problem posed, therefore, is to change the ratio of demand for two goods by a specified amount, while maintaining the same level of utility.

The ratio of the two components is given by the following expression using equation (A-9) as the starting point:

$$R = \frac{\alpha_1 \lambda_2 P_2^\sigma}{\alpha_2 \lambda_1 P_1^\sigma}$$

The idea is to move the initial ratio,  $R_{t-1}$  to  $R_t$  by  $tw$  percent.

$$\frac{R_t}{R_{t-1}} = (1 + tw_t)$$

while holding  $U$  constant. The two expressions above imply that the preference shifters, given by the  $\pi$  parameters, are linked via the following expression:

$$1 + \pi_2 = (1 + \pi_1) (1 + tw)^{1/\sigma} \tag{A-13}$$

Given equation (A-12), holding  $U$  constant is equivalent to holding the price index,  $P^c$ , constant as well (for a fixed aggregate volume). Thus we can solve the following equation for the parameter  $\pi_1$ :

$$\begin{aligned} (P_{t-1}^c)^{-\sigma} &= \alpha_1 (P_{1,t-1} \lambda_{1,t-1})^{-\sigma} + \alpha_2 (P_{2,t-1} \lambda_{2,t-1})^{-\sigma} \\ &= \alpha_1 (P_{1,t-1} \lambda_{1,t-1} (1 + \pi_{1,t}))^{-\sigma} + \alpha_2 (P_{2,t-1} \lambda_{2,t-1} (1 + \pi_{2,t}))^{-\sigma} \\ &= \alpha_1 (P_{1,t-1} \lambda_{1,t-1} (1 + \pi_{1,t}))^{-\sigma} + \alpha_2 \left( P_{2,t-1} \lambda_{2,t-1} (1 + \pi_{1,t}) (1 + tw)^{1/\sigma} \right)^{-\sigma} \\ &= (P_t^c)^{-\sigma} \end{aligned}$$

The  $\pi$  variables represent the growth (either positive or negative) that will be applied to the preference parameters under the assumption of utility-preserving preference shifts. This formula can be written in terms of the initial volume shares,  $r_i = X_i/V$ , simplified and re-arranged to yield:

$$(1 + \pi_1)^\sigma = r_1 + \frac{r_2}{1 + tw}$$

and when re-inserted in equation (A-13) we get:

$$(1 + \pi_2)^\sigma = \left( r_1 + \frac{r_2}{1 + tw} \right) (1 + tw)$$

The final formulas for the two twist parameters only depend on the initial volume shares, the substitution elasticity and the level of the 'twist':

$$\pi_1 = \left( r_1 + \frac{r_2}{1 + tw} \right)^{1/\sigma} - 1 \tag{A-14}$$

$$\pi_2 = (r_1 (1 + tw) + r_2)^{1/\sigma} - 1 \tag{A-15}$$

It is possible to generalize these formulas by partitioning the set of CES components into two sets—a set indexed by 1 that is the target set, and a set indexed by 2 that is the complement. For example, think of a set of electricity technologies that includes conventional and advanced. It is possible then to provide the same twist to all of the new technologies relative to the conventional technologies. The only change in the formulas above is that the volume share variable for the single component is replaced by the sum of the volume shares for the bundle of components:

$$\lambda_{1,t} = (1 + \pi_{1,t}) \lambda_{1,t-1} = \left[ \sum_{i \in 1} r_{i,t-1} + \frac{\sum_{i \in 2} r_{i,t-1}}{1 + tw_t} \right]^{1/\sigma} \lambda_{1,t-1}$$

$$\lambda_{2,t} = (1 + \pi_{2,t}) \lambda_{2,t-1} = \left[ \sum_{i \in 1} (1 + tw_t) r_{i,t-1} + \sum_{i \in 2} r_{i,t-1} \right]^{1/\sigma} \lambda_{2,t-1}$$

### Converting to percent differences

The  $\pi$  factors reflect a percentage change in the relevant productivity factors for each of the components. Using a Taylor series approximation, the formulas above can be converted to a linear equation that is used by the Monash-style models. For the first component, we have:

$$\pi_1 = F(tw) = \left[ r_1 + \frac{r_2}{1 + tw} \right]^{1/\sigma} - 1 \approx F(0) + tw.F'(0) = -tw \frac{r_2}{\sigma}$$

For the second component we have:

$$\pi_2 = F(tw) = [r_1 (1 + tw) + r_2]^{1/\sigma} - 1 \approx F(0) + tw.F'(0) = tw \frac{r_1}{\sigma}$$

# Appendix B

## The demand systems

This appendix describes in full detail the consumer demand systems available in the ENVISAGE Model. The first section derives the main relations for the extended linear expenditure system (ELES). The second section describes the constant differences in elasticity (CDE) demand system.

### B.1 The extended linear expenditure system (ELES)

Many models assume separability in household decision making between saving and current consumption. Lluch and Howe<sup>1</sup> introduced a relatively straightforward extension of the LES consumer demand function to include the saving decision simultaneously with the allocation of income on goods and services, this has become known as the extended linear expenditure system or the ELES. The ELES is based on consumers maximizing their intertemporal utility between a bundle of current consumption and an expected future consumption bundle represented in the form of savings.

#### B.1.1 Basic formulation

The utility function of the ELES has the following form:

$$u = \prod_i (x_i - \theta_i)^{\mu_i} \left( \frac{S}{P^s} \right)^{\mu_s} \quad (\text{B-1})$$

with

$$\sum_i \mu_i + \mu_s = 1 \quad (\text{B-2})$$

where  $u$  is utility,  $x$  is the vector of consumption goods,  $S$  is household saving (in value),  $P^s$  is the price of saving, and  $\mu$  and  $\theta$  are ELES parameters.

The consumer solves the following problem:

$$\max \prod_i (x_i - \theta_i)^{\mu_i} \left( \frac{S}{P^s} \right)^{\mu_s}$$

subject to

---

<sup>1</sup> See Lluch (1973) and Howe (1975).

$$\sum_{i=1}^n p_i x_i + S = Y$$

where  $p$  is the vector of consumer prices, and  $Y$  is disposable income. The demand functions are:

$$x_i = \theta_i + \frac{\mu_i}{p_i} \left( Y - \sum_{j=1}^n p_j \theta_j \right) \quad (\text{B-3})$$

$$S = \mu_s \left( Y - \sum_{j=1}^n p_j \theta_j \right) = Y - \sum_{j=1}^n p_j x_j \quad (\text{B-4})$$

The term in parentheses is sometimes called supernumerary income, i.e. it is the income that remains after subtracting total expenditures on the so-called subsistence (or floor) expenditures as represented by the  $\theta$  parameter. The parameter  $\mu$  then represents the marginal budget share out of supernumerary income.

### B.1.2 ELES elasticities

From the demand equation we can derive the income and price elasticities:

$$\eta_i = \frac{\mu_i Y}{p_i x_i} = \frac{\mu_i}{s_i} \quad \eta_s = \frac{\mu_s Y}{S} = \frac{\mu_s}{s} \quad (\text{B-5})$$

$$\varepsilon_i = \frac{\theta_i (1 - \mu_i)}{x_i} - 1 \quad \varepsilon_s = -1 \quad (\text{B-6})$$

$$\varepsilon_{ij} = -\frac{\mu_i p_j \theta_j}{p_i x_i} = -\frac{\mu_i p_j \theta_j}{s_i Y} \quad \varepsilon_{sj} = -\frac{\mu_s p_j \theta_j}{s Y} = -\frac{p_j \theta_j}{Y^*} \quad (\text{B-7})$$

where  $s$  is the average propensity to save. Note that the matrix of elasticities can be collapsed to a single formula using the Kronecker factor:

$$\varepsilon_{ij} = -\frac{\mu_i p_j \theta_j}{p_i x_i} - \delta_{ij} \frac{p_i x_i - p_i \theta_i}{p_i x_i} = -\frac{\mu_i}{s_i Y} [\delta_{ij} Y^* + p_j \theta_j] = -\eta_i \left[ \delta_{ij} \frac{Y^*}{Y} + \frac{p_j \theta_j}{Y} \right] \quad (\text{B-8})$$

The last expression shows that there is clear linkage between the income and price elasticities. At the limit, when income is much larger than supernumerary income, the two are virtually identical in levels (with opposite signs).

### B.1.3 Welfare

With the addition of saving, the indirect utility function is given by:

$$v(p, Y) = \prod_i \left( \frac{\mu_i}{p_i} Y^* \right)^{\mu_i} \left( \frac{\mu_s}{P^s} Y^* \right)^{\mu_s} \quad (\text{B-9})$$

or

$$v(p, Y) = \frac{Y^*}{P} \quad (\text{B-10})$$

where



$$P = \prod_i \left( \frac{p_i}{\mu_i} \right)^{\mu_i} \left( \frac{P^s}{\mu_s} \right)^{\mu_s}$$

The expenditure function is derived by minimizing the cost of achieving a given level of utility,  $u$ . It is set-up as:

$$\min \sum_{i=1}^n p_i x_i + S$$

subject to

$$\prod_i (x_i - \theta_i)^{\mu_i} \left( \frac{S}{P^s} \right)^{\mu_s} = u$$

The final expression for the expenditure function is:

$$E(p, u) = \sum_{i=1}^n p_i \theta_i + uP \tag{B-11}$$

where  $P$ , the aggregate price index (including the price of savings) is defined as above.

### B.1.4 Calibration

Calibration of the ELES uses the budget share information from the base SAM, including the household saving share. Typically, calibration uses income elasticities for all of the  $n$  commodities represented in the demand system and uses equation (B-5) to derive the marginal budget shares,  $\mu_i$ . This procedure leads to a residual income elasticity, which in this case is the income elasticity of saving. The derived savings income elasticity may be implausible, in which case adjustments need to be made to individual income elasticities for the goods, or adjustments can be made on the group of goods, assuming some target for the savings income elasticity.

The first step is therefore to calculate the marginal budget shares using the average budget shares and the initial income elasticity estimates.

$$\mu_i = \frac{\eta_i P_i x_i}{Y} = \eta_i s_i$$

The savings marginal budget share is derived from the consistency requirement that the marginal budget shares sum to 1:

$$\mu_s = 1 - \sum_{i=1}^n \mu_i$$

Assuming this procedure leads to a plausible estimate for the savings income elasticity, the next step is to calibrate the subsistence minima,  $\theta$ . This can be done by seeing that the demand equations, (B-3), are linear in the  $\theta$  parameters. Note that in the case of the ELES the system of equation are of full rank because the  $\mu_i$  parameters do not sum to 1 (over the  $n$  commodities)—they only sum to 1 including the marginal saving share.<sup>2</sup> This may lead to calibration problems if the propensity to save is 0, which may be the case in some SAMs with poor households. The linear system can be written as:

<sup>2</sup> Note that the calibration and the setup of the ELES assume explicitly that the minimal expenditure on savings is zero.

$$C = I\theta + MY - M\Pi\theta$$

where  $I$  is an  $n \times n$  identity matrix,  $M$  is a diagonal matrix with  $\mu_i/P_i$  on the diagonal, and  $\Pi$  is a matrix, where each row is identical, each row being the transpose of the price vector. The above system of linear equations can be solved via matrix inversion for the parameter  $\theta$ :

$$\theta = A^{-1}C^*$$

where

$$\begin{aligned} A &= I - M\Pi \\ C^* &= C - MY \end{aligned}$$

The matrices  $A$  and  $C^*$  are defined by:

$$\begin{aligned} A = [a_{ij}] &= \left[ \delta_{ij} - \mu_i \frac{p_j}{p_i} \right] = \begin{cases} 1 - \mu_i & \text{if } i = j \\ -\mu_i \frac{p_j}{p_i} & \text{if } i \neq j \end{cases} \\ C^* = [c_i] &= x_i - \frac{\mu_i Y}{p_i} \end{aligned}$$

The  $A$  and  $C^*$  matrices are simplified if the price vector is initialized at 1:

$$\begin{aligned} A = [a_{ij}] &= \begin{cases} 1 - \mu_i & \text{if } i = j \\ -\mu_i & \text{if } i \neq j \end{cases} \\ C^* = [c_i] &= x_i - \mu_i Y \end{aligned}$$

In GAMS one could invert the system of equations embodied in equation (B-3) directly by solving for the endogenous  $\theta$  while holding all of the other variables and parameters fixed.

## B.2 An Implicitly Direct Additive Demand System (AIDADS)

Many commonly used utility functions typically exhibit poor Engel behavior—particularly in a dynamic framework. The CDE utility function, popularized by the GTAP model (see Hertel (1997)), has relatively constant income elasticities. The LES utility function has even worse behavior, as in the absence of any shifts in the underlying parameters, the LES converges relatively quickly to a Cobb-Douglas utility function as rapidly rising consumption tends to dominate the floor consumption parameters, even when adjusting the latter to take into account population growth. Rimmer and Powell (see Rimmer and Powell (1992b), Rimmer and Powell (1992a) and Rimmer and Powell (1996)) examine an extension to the standard LES demand system that in effect allows the marginal propensity to consumer parameter to be driven by changes in utility. Their utility function has been called An Implicitly Direct Additive Demand System, or AIDADS. The LES function is a special case of the AIDADS system where the marginal propensity variable is constant. This extension allows for more complex demand behavior, as well as providing better validation for observed changes in consumption patterns.<sup>3</sup>

<sup>3</sup> AIDADS has also been explored in the context of the GTAP model, see for example Yu et al. (2003).

### B.2.1 Basic formulation

AIDADS starts with the implicitly additive utility function given by:

$$\sum_i U_i(x_i, u) \equiv 1 \quad (\text{B-12})$$

Assume the following functional form for the utility function:

$$U_i = \mu_i \ln \left( \frac{x_i - \theta_i}{Ae^u} \right) \quad (\text{B-13})$$

where

$$\mu_i = \frac{\alpha_i + \beta_i G(u)}{1 + G(u)} \quad (\text{B-14})$$

with the restrictions

$$\begin{aligned} \sum_i \alpha_i &= \sum_i \beta_i = 1 \\ 0 &\leq \alpha_i \leq 1 \\ 0 &\leq \beta_i \leq 1 \\ \theta_i &< x_i \end{aligned}$$

Cost minimization implies the following:

$$\min \sum_i p_i x_i$$

subject to

$$\sum_i \mu_i \ln \left( \frac{x_i - \theta_i}{Ae^u} \right) \equiv 1 \quad (\text{B-15})$$

The first order conditions lead to:

$$\lambda \frac{\partial U_i}{\partial x_i} = p_i = \lambda \frac{\mu_i}{x_i - \theta_i} \Rightarrow \lambda \mu_i = p_i x_i - p_i \theta_i \quad (\text{B-16})$$

Taking the sum over  $i$  and using the fact that the  $\mu_i$  sum to unity implies:

$$\lambda = \sum_i p_i x_i - \sum_i p_i \theta_i = Y - \sum_i p_i \theta_i = Y^* \quad (\text{B-17})$$

where  $Y$  is aggregate expenditure, and  $Y^*$ , sometimes referred to as supernumerary income, is residual expenditure after subtracting total expenditure on the so-called subsistence minima,  $\theta$ .

Re-inserting equation (B-17) into (B-16) yields the consumer demand equations:

$$x_i = \theta_i + \frac{\mu_i}{p_i} Y^* = \theta_i + \frac{\mu_i}{p_i} \left[ Y - \sum_j p_j \theta_j \right] \quad (\text{B-18})$$

Equation (B-18) is virtually identical to the LES demand equation. Similar to the linear expenditure system (LES), demand is the sum of two components—a subsistence minimum,  $\theta$ , and

a share,  $\mu$ , of supernumerary income. Unlike the LES, the share parameter,  $\mu$ , is not constant, but depends on the level of utility itself. AIDADS collapses to the LES if each  $\alpha$  parameter is equal to the corresponding  $\beta$  parameter, with the ensuing function of utility,  $G(u)$ , dropping from equation (B-14).

### B.2.2 Elasticities

This section develops the main expressions for the income and price elasticities. These formulas will be needed to calibrate the initial parameters of the AIDADS function.

#### Income elasticities

To derive further properties of AIDADS requires specifying a functional form for  $G(u)$ . Rimmer and Powell (1996) propose the following:

$$G(u) = e^u \quad (\text{B-19})$$

The first step is to calculate the marginal budget share,  $\rho$ , defined as:

$$\rho_i = p_i \frac{\partial x_i}{\partial Y}$$

The following expression can be derived from equation (B-18):

$$\frac{\partial x_i}{\partial Y} = \frac{Y^*}{p_i} \frac{\partial \mu_i}{\partial Y} + \frac{\mu_i}{p_i} \frac{\partial Y^*}{\partial Y} = \frac{Y^*}{p_i} \frac{\partial \mu_i}{\partial u} \frac{\partial u}{\partial Y} + \frac{\mu_i}{p_i}$$

Thus:

$$\rho_i = \mu_i + Y^* \frac{\partial \mu_i}{\partial u} \frac{\partial u}{\partial Y} \quad (\text{B-20})$$

Expression (B-20) can be expanded in two steps—first evaluating the partial derivative of the share variable,  $\mu$ , with respect to utility, and then the more difficult calculation of the partial derivative of  $u$  with respect to income. The marginal share formula is:

$$\mu_i = \frac{\alpha_i + \beta_i e^u}{1 + e^u}$$

Its derivative is:

$$\frac{\partial \mu_i}{\partial u} = \frac{(1 + e^u)(\beta_i e^u) - (\alpha_i + \beta_i e^u)e^u}{(1 + e^u)^2} = \frac{e^u(\beta_i - \alpha_i)}{(1 + e^u)^2} \quad (\text{B-21})$$

Utility and income are combined in implicit form and thus we will invoke the implicit function theorem to calculate the partial derivative of  $u$  with respect to  $Y$ . First, insert equation (B-18) into equation (B-15):

$$\sum_i \mu_i \ln \left( \frac{x_i - \theta_i}{Ae^u} \right) = \sum_i \mu_i \ln \left( \frac{\mu_i Y^*}{Ae^u p_i} \right) = 1$$

Expanding the latter expression yields:

$$f(u, Y) = \sum_i \mu_i \ln \left( \frac{\mu_i}{p_i} \right) + \ln(Y^*) - \ln(A) - u = 1 \quad (\text{B-22})$$

which provides the implicit relation between  $Y$  and  $u$ . The implicit function theorem states the following:

$$\frac{\partial u}{\partial Y} = -\frac{\partial f}{\partial Y} \left[ \frac{\partial f}{\partial u} \right]^{-1} \quad (\text{B-23})$$

The partial derivative of  $f$  with respect to  $Y$  is simply:

$$\frac{\partial f}{\partial Y} = \frac{1}{Y^*} \quad (\text{B-24})$$

The partial derivative of  $f$  with respect to  $u$  is:

$$\begin{aligned} \frac{\partial f}{\partial u} &= -1 + \sum_i \left[ \frac{\partial \mu_i}{\partial u} \ln \left( \frac{\mu_i}{p_i} \right) + \mu_i \frac{p_i}{\mu_i} \frac{\partial \mu_i}{\partial u} \right] \\ &= -1 + \frac{e^u}{(1+e^u)^2} \sum_i \left[ \left( \ln \left( \frac{\mu_i}{p_i} \right) + 1 \right) (\beta_i - \alpha_i) \right] \\ &= \frac{e^u}{(1+e^u)^2} \left[ \sum_i (\beta_i - \alpha_i) \ln (x_i - \theta_i) - \frac{(1+e^u)^2}{e^u} \right] \\ &= \frac{e^u}{(1+e^u)^2} \Omega^{-1} \end{aligned} \quad (\text{B-25})$$

where

$$\Omega = \left[ \sum_i (\beta_i - \alpha_i) \ln (x_i - \theta_i) - \frac{(1+e^u)^2}{e^u} \right]^{-1} \quad (\text{B-26})$$

The second line uses equation (B-21). In the third line, equation (B-18) substitutes for the expression in the logarithm, and the adding up constraint allows for the deletion of non-indexed variables. Substituting equations (B-24) and (B-25) into equation (B-23) yields:

$$\frac{\partial u}{\partial Y} = -\frac{\Omega}{Y^*} \frac{(1+e^u)^2}{e^u} \quad (\text{B-27})$$

Substituting equations (B-21) and (B-27) into equation (B-20) yields the following expression for  $\rho$ :

$$\rho_i = \mu_i - (\beta_i - \alpha_i) \Omega$$

The income elasticities are derived from the following expression:

$$\eta_i = \frac{\partial x_i}{\partial Y} \frac{Y}{x_i} = \frac{\partial x_i}{\partial Y} \frac{Y}{x_i} \frac{p_i}{p_i} = \frac{\rho_i}{s_i}$$

where  $s_i$  is the average budget share:

$$s_i = \frac{p_i x_i}{Y} = \frac{p_i \theta_i}{Y} + \mu_i \frac{Y^*}{Y} = \mu_i + \left( \frac{p_i \theta_i - \mu_i \sum_j p_j \theta_j}{Y} \right)$$

Thus the income elasticity,  $\eta$ , is equal to the ratio of the marginal budget share,  $\rho$ , and the average budget share,  $s$ . Finally, equation (B-28) describes one formulation of the income elasticity:

$$\eta_i = \frac{\mu_i - (\beta_i - \alpha_i) \Omega}{s_i} \quad (\text{B-28})$$

### Price elasticity

The matrix of substitution elasticities is identical to the expression for the LES and has the form:

$$\sigma_{ij} = [\mu_j - \delta_{ij}] \frac{\mu_i Y^*}{s_i s_j Y} \quad (\text{B-29})$$

where  $\delta$  is the Kronecker delta:

$$\delta_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$$

It is clear that the matrix is symmetric. The matrix of substitution elasticities is also equal to:

$$\sigma_{ij} = [\mu_j - \delta_{ij}] \frac{\mu_i Y^*}{s_i s_j Y} = \frac{(x_i - \theta_i)}{x_i} \frac{(x_j - \theta_j)}{x_j} \frac{Y}{Y^*} - \frac{\delta_{ij}}{s_j} \frac{(x_i - \theta_i)}{x_i}$$

The compensated demand elasticities derive from the following:

$$\xi_{ij} = s_j \sigma_{ij} = [\mu_j - \delta_{ij}] \frac{\mu_i Y^*}{s_i Y} \quad (\text{B-30})$$

Finally, the matrix of uncompensated demand elasticities is given by:

$$\varepsilon_{ij} = \xi_{ij} - s_j \eta_i = [\mu_j - \delta_{ij}] \frac{\mu_i Y^*}{s_i Y} - s_j \eta_i \quad (\text{B-31})$$

The uncompensated demand elasticities can also be written as:

$$\varepsilon_{ij} = -\frac{\mu_i}{s_i Y} [p_j \theta_j + \delta_{ij} Y^*] + \frac{s_j}{s_i} (\beta_i - \alpha_i) \Omega \quad (\text{B-32})$$

The first term on the right-hand side is always negative. The second term differs from the LES expression for the uncompensated demand elasticities.<sup>4</sup> We can see from expression (B-32) that the AIDADS specification allows for both gross complementarity and substitution. As well, it allows for luxury goods, i.e. positive own-price demand elasticities should the second term be positive and greater than the first term.

### B.2.3 Implementation

Implementation of AIDADS is somewhat more complicated than the LES since the marginal propensity to consume out of supernumerary income is endogenous, and utility is defined implicitly. The following four equations are needed for model implementation:

$$Y^* = Y - \sum_i p_i \theta_i \quad (\text{B-33})$$

$$x_i = \theta_i + \frac{\mu_i}{p_i} Y^* \quad (\text{B-34})$$

$$\mu_i = \frac{\alpha_i + \beta_i e^u}{1 + e^u} \quad (\text{B-35})$$

$$u = \sum_i \mu_i \ln(x_i - \theta_i) - 1 - \ln(A) \quad (\text{B-36})$$

<sup>4</sup> Recall that for the LES, the  $\alpha$  and  $\beta$  terms are equal and thus the second term drops.

Equations (B-33) and (B-34) are identical to their LES and ELES counterparts.<sup>5</sup> Equation (B-35) determines the level of the marginal propensity to consume out of supernumerary income,  $\mu$ , which is a constant in the case of the LES (ELES). It requires however the calculation of the utility level,  $u$ , which is defined in equation (B-36).

### B.2.4 Calibration

[To be updated] Calibration requires more information than the LES. Where the LES has  $2n$  parameters to calibrate (subject to consistency constraints), AIDADS has  $3n$  parameters (less the consistency requirements)— $\alpha$ ,  $\beta$  and  $\theta$ . The calibration system includes equations (B-33) through (B-36) which have  $2 + 2n$  endogenous variables ( $Y^*$ ,  $\theta$ ,  $\mu$ , and  $A$ ). There are no equations for calibrating the  $\alpha$  and  $\beta$  parameters. If we have knowledge of the income elasticities, we can add the following equations:

$$\Psi = \frac{1}{\Omega} = \left[ \sum_i (\beta_i - \alpha_i) \ln(x_i - \theta_i) - \frac{(1 + e^u)^2}{e^u} \right] \quad (\text{B-37})$$

$$\eta_i = \frac{\rho_i}{s_i} = \frac{\mu_i - (\beta_i - \alpha_i)\Omega}{s_i} = \frac{\mu_i}{s_i} - \frac{(\beta_i - \alpha_i)}{s_i\Psi} \quad (\text{B-38})$$

There are an additional  $1 + n$  equations, solving for  $\Psi$  and  $\alpha$ . There is need for an additional  $n$  equations. Assuming we have knowledge of at least  $n$  price elasticities, for example the own-price elasticities, we can add the following equation:

$$\varepsilon_{ii} = -\frac{\mu_i}{s_i Y} [p_i \theta_i + Y^*] + (\beta_i - \alpha_i)\Omega \quad (\text{B-39})$$

The  $\alpha$  and  $\beta$  parameters are not independent, the following restrictions must hold:

$$\sum_i \alpha_i = 1 \quad (\text{B-40})$$

$$\sum_i \beta_i = 1 \quad (\text{B-41})$$

The system is under-determined, there are  $5 + 4n$  equations and  $3 + 4n$  variables. One solution, is to make the own-price elasticities endogenous. In this case, we are adding  $n$  variables, but then the system is over-determined. We can minimize a loss function with respect to the price elasticities:

$$L = \sum_i (\varepsilon_i - \varepsilon_i^0)^2$$

where  $\varepsilon^0$  represents an initial guess of the own-price elasticities and the calibration algorithm will calculate the endogenous  $\varepsilon$  in order to minimize the loss function, subject to constraints (B-37) through (B-41) and the model equations (B-33) through (B-36). The exogenous parameters in the calibration procedure include  $p$ ,  $x$ ,  $s$ ,  $Y$ ,  $\eta$ ,  $\varepsilon^0$  and  $u$ .

<sup>5</sup> Though the definition of  $Y$  includes savings in the case of the ELES.

### B.3 Constant differences in elasticity (CDE) demand system

The *Constant Difference of Elasticities* (CDE) function is a generalization of the CES function, but it allows for more flexibility in terms of substitution effects across goods and for non-homotheticity.<sup>6</sup> The starting point is an implicitly additive indirect utility function (see [Hanoch \(1975\)](#)) from which we can derive demand using Roy's identity (and the implicit function theorem).

#### B.3.1 General form

A dual approach is used to determine the properties of the CDE function. The indirect utility function is defined implicitly via the following expression:

$$V(p, u, Y) = \sum_{i=1}^n \alpha_i u^{e_i b_i} \left( \frac{p_i}{y} \right)^{b_i} \equiv 1 \quad (\text{B-42})$$

where  $p$  is the vector of commodity prices,  $u$  is (per capita) utility and  $y$  is per capita income. Using Roy's identity and the implicit function theorem<sup>7</sup> we can derive demand,  $x$ , where  $v$  is the indirect utility function (defined implicitly):

$$x_i = -\frac{\partial v / \partial p_i}{\partial v / \partial Y} = -\left( \frac{\partial V / \partial p_i}{\partial V / \partial u} \right) / \left( \frac{\partial V / \partial Y}{\partial V / \partial u} \right) = -\left( \frac{\partial V / \partial p_i}{\partial V / \partial Y} \right) \quad (\text{B-43})$$

This then leads to the following demand function:

$$x_i = \frac{\alpha_i b_i u^{e_i b_i} \left( \frac{p_i}{y} \right)^{b_i - 1}}{\sum_j \alpha_j b_j u^{e_j b_j} \left( \frac{p_j}{y} \right)^{b_j}} \quad (\text{B-44})$$

Implementation is easier if we define the following variable:

$$\theta_i = \alpha_i b_i u^{e_i b_i} \left( \frac{p_i}{y} \right)^{b_i} \quad (\text{B-45})$$

Then the budget shares can be expressed as:

$$s_i = \frac{\theta_i}{\sum_j \theta_j} \quad (\text{B-46})$$

and the demand expression is:

$$x_i = \frac{s_i}{p_i} y \quad (\text{B-47})$$

Implementation also requires evaluating  $u$ . This can be done by implementing equation (B-42) and inserting the expression for  $\theta$ :

$$\sum_{i=1}^n \frac{\theta_i}{b_i} \equiv 1 \quad (\text{B-48})$$

<sup>6</sup> More detailed descriptions of the CDE can be found in [Hertel, Preckel, and Tsigas \(1991\)](#), [Surry \(1993\)](#) and [Hertel \(1997\)](#).

<sup>7</sup> See [Varian \(1992\)](#), p. 109.



### B.3.2 Elasticities

In order to calibrate the CDE system, it is necessary to derive the demand and income elasticities of the CDE. The algebra is tedious, but straightforward.

The own-price elasticity is given by the following:

$$\varepsilon_i = \frac{\partial x_i}{\partial p_i} \frac{p_i}{x_i} = \frac{s_i \left[ \sum_j s_j e_j b_j - e_i b_i \right]}{\sum_j s_j b_j} + b_i (1 - s_i) - 1 \quad (\text{B-49})$$

In deriving the elasticity, we make use of the following formula that defines the elasticity of utility with respect to price (and again makes use of the implicit function theorem):

$$\frac{\partial u}{\partial p_i} \frac{p_i}{u} = -\frac{p_i}{u} \left( \frac{\partial V}{\partial p_i} \right) / \left( \frac{\partial V}{\partial u} \right) = -\frac{s_i}{\sum_j s_j e_j} \quad (\text{B-50})$$

The price elasticity of utility is approximately the value share of the respective demand component as long as the weighted sum of the expansion parameters,  $e$ , is close to unity. The value (or budget) share is defined in the next equation:

$$s_i = \frac{p_i x_i}{y} \quad (\text{B-51})$$

Letting  $\sigma_i = 1 - b_i$  (or  $b_i = 1 - \sigma_i$ ), we can also write:

$$\varepsilon_i = s_i \left[ \sigma_i - \frac{e_i(1 - \sigma_i)}{\sum_j s_j e_j} - \frac{\sum_j s_j e_j \sigma_j}{\sum_j s_j e_j} \right] - \sigma_i \quad (\text{B-52})$$

With  $\sigma$  uniform, we also have:

$$\varepsilon_i = -\frac{s_i e_i (1 - \sigma)}{\sum_j s_j e_j} - \sigma \quad (\text{B-53})$$

With both  $e$  and  $\sigma$  uniform, the formula simplifies to:

$$\varepsilon_i = -s_i(1 - \sigma) - \sigma = \sigma(s_i - 1) - s_i \quad (\text{B-54})$$

Equation (B-54) reflects the own-price elasticity for the standard CES utility function. Finally, with  $e$  uniform but not  $\sigma$ , we have:

$$\varepsilon_i = s_i \left[ 2\sigma_i - 1 - \sum_j s_j \sigma_j \right] - \sigma_i \quad (\text{B-55})$$

The derivation of the cross elasticities is almost identical and will not be carried out here. Combining both the own-and cross price elasticities, the matrix of substitution elasticities takes

the following form where we use the Kronecker product,  $\delta$ :<sup>8</sup>

$$\varepsilon_{ij} = s_j \left[ -b_j - \frac{e_i b_i}{\sum_k s_k e_k} + \frac{\sum_k s_k e_k b_k}{\sum_k s_k e_k} \right] + \delta_{ij}(b_i - 1) \quad (\text{B-56})$$

Again, we replace  $b$  by  $1 - \sigma$ , to get:

$$\varepsilon_{ij} = s_j \left[ \sigma_j - \frac{e_i(1 - \sigma_i)}{\sum_k s_k e_k} - \frac{\sum_k s_k e_k \sigma_k}{\sum_k s_k e_k} \right] - \delta_{ij} \sigma_i \quad (\text{B-57})$$

For uniform  $\sigma$ , equation (B-57) takes the form:

$$\varepsilon_{ij} = -\frac{e_i s_j (1 - \sigma)}{\sum_k s_k e_k} - \delta_{ij} \sigma \quad (\text{B-58})$$

And with a uniform  $\sigma$  and  $e$ , i.e. the CES assumption, we have:

$$\varepsilon_{ij} = -s_j(1 - \sigma) - \delta_{ij} \sigma = \sigma(s_j - \delta_{ij}) - s_j \quad (\text{B-59})$$

Finally, for a uniform  $e$  only, the matrix of elasticities is:

$$\varepsilon_{ij} = s_j \left[ \sigma_j - (1 - \sigma_i) - \sum_k s_k \sigma_k \right] - \delta_{ij} \sigma_i \quad (\text{B-60})$$

The income elasticities are derived in a similar fashion:

$$\eta_i = \frac{\partial x_i}{\partial Y} \frac{Y}{x_i} = \frac{1}{\sum_k s_k e_k} \left[ e_i b_i - \sum_k s_k e_k b_k \right] - (b_i - 1) + \sum_k b_k s_k \quad (\text{B-61})$$

For this, we need the elasticity of utility with respect to income:

$$\frac{\partial u}{\partial Y} \frac{Y}{u} = -\frac{Y}{u} \left( \frac{\partial V}{\partial Y} \right) / \left( \frac{\partial V}{\partial u} \right) = \frac{1}{\sum_k s_k e_k} \quad (\text{B-62})$$

Note that for a uniform and unitary  $e$ , the income elasticity of utility is 1.

Replacing  $b$  with  $1 - \sigma$ , equation (B-61) can be re-written to be:

$$\eta_i = \frac{1}{\sum_k s_k e_k} \left[ e_i(1 - \sigma_i) + \sum_k s_k e_k \sigma_k \right] + \sigma_i - \sum_k s_k \sigma_k \quad (\text{B-63})$$

With a uniform  $\sigma$ , the income elasticity becomes:

$$\eta_i = \frac{1}{\sum_k s_k e_k} \left[ e_i(1 - \sigma) + \sigma \sum_k s_k e_k \right] = \frac{e_i(1 - \sigma)}{\sum_k s_k e_k} + \sigma \quad (\text{B-64})$$

With  $e$  uniform, the income elasticity is unitary, irrespective of the values of the  $\sigma$  parameters.

<sup>8</sup>  $\delta$  takes the value of 1 along the diagonal (i.e. when  $i = j$ ) and the value 0 off-diagonal (i.e. when  $i \neq j$ ).

From the Slutsky equation, we can calculate the compensated demand elasticities:

$$\xi_{ij} = \varepsilon_{ij} + s_j \eta_i = -\delta_{ij} \sigma_i + s_j \left[ \sigma_j + \sigma_i - \sum_k s_k \sigma_k \right] \quad (\text{B-65})$$

The cross-Allen partial elasticities are equal to the compensated demand elasticities divided by the share:

$$\sigma_{ij}^a = \sigma_j + \sigma_i - \sum_k s_k \sigma_k - \delta_{ij} \sigma_i / s_j \quad (\text{B-66})$$

It can be readily seen that the difference of the partial elasticities is constant, hence the name of *constant difference in elasticities*.

$$\sigma_{ij}^a - \sigma_{il}^a = \sigma_j - \sigma_l \quad (\text{B-67})$$

With a uniform  $\sigma$ , we revert back to the standard CES where there is equivalence between the CES substitution elasticity and the cross-Allen partial elasticity:

$$\sigma_{ij}^a = \sigma \quad (\text{B-68})$$

### B.3.3 Calibration of the CDE

Calibration assumes that we know the budget shares, the own uncompensated demand elasticities and the income elasticities. The weighted sum of the income elasticities must equal 1, so the first step in the calibration procedure is to make sure Engel's law holds. One alternative is to fix some (or none) of the income elasticities and re-scale the others using least squares. The problem is to minimize the following objective function:

$$\sum_{i \in \Omega} (\eta_i - \eta_i^0)^2$$

subject to

$$\sum_{i \in \Omega} s_i \eta_i = 1 - \sum_{i \notin \Omega} s_i \eta_i$$

where the set  $\Omega$  contains all sectors where the income elasticity is not fixed, i.e. its complement contains those sectors with fixed income elasticities. The solution is:

$$\eta_i = \eta_i^0 + s_i \frac{1 - \sum_{j \notin \Omega} s_j \eta_j - \sum_{j \in \Omega} s_j \eta_j^0}{\sum_{j \in \Omega} s_j^2} \quad \forall i \in \Omega$$

Calibration of the  $\sigma$  parameters is straightforward given the own elasticities and the input value shares. The first step is to calculate the Allen partial elasticities, these are simply the income elasticity adjusted by the own elasticities divided by the budget shares:

$$\sigma_{ii}^a = \eta_i + \frac{\varepsilon_{ii}}{s_i} \quad (\text{B-69})$$

Next, equation (B-66) is setup in matrix form:

$$\sigma_{ii}^a = A\sigma_i \quad (\text{B-70})$$

where the matrix  $A$  has the form:

$$A = \begin{bmatrix} 2 - \frac{1}{s_1} - s_1 & -s_2 & \dots & -s_n \\ -s_1 & 2 - \frac{1}{s_2} - s_2 & \dots & -s_n \\ \vdots & \vdots & \ddots & \vdots \\ -s_1 & -s_2 & \dots & 2 - \frac{1}{s_n} - s_n \end{bmatrix} \quad (\text{B-71})$$

or each element of  $A$  has the following formula:

$$a_{ij} = \delta_{ij}(2 - 1/s_i) - s_j$$

We can then solve for  $\sigma$  (and back-out the  $b$  parameters):

$$\sigma_i = A^{-1}\sigma_{ii}^a \quad (\text{B-72})$$

There is nothing which guarantees the consistency of the calibrated  $\sigma$  parameters, which are meant to be positive. The calculation of the  $\sigma$  parameters depends only on the budget shares and the own-price uncompensated elasticities. If the calibrated  $\sigma$  parameters are not all positive, one could modify the elasticities until consistency is achieved. In practice, problems have occurred when a sector's budget share dominates total expenditure.

The  $e$  parameters are derived from Equation (B-63) and normalizing them so that their share weighted sum is equal to 1. Equation (B-63) can then be converted to matrix form and inverted:

$$B = \begin{bmatrix} s_1\sigma_1 + (1 - \sigma_1) & s_2\sigma_2 & \dots & s_n\sigma_n \\ s_1\sigma_1 & s_2\sigma_2 + (1 - \sigma_2) & \dots & s_n\sigma_n \\ \vdots & \vdots & \ddots & \vdots \\ s_1\sigma_1 & s_2\sigma_2 & \dots & s_n\sigma_n + (1 - \sigma_n) \end{bmatrix} \quad (\text{B-73})$$

or

$$b_{ij} = s_j\sigma_j + \delta_{ij}(1 - \sigma_i)$$

Then the  $e$  parameters are derived from matrix inversion:

$$e_i = B^{-1}C_i = B^{-1} \left( \eta_i - \sigma_i + \sum_k s_k\sigma_k \right) \quad (\text{B-74})$$

Calibration of the  $\alpha$  parameters is based on equations (B-42) and (B-44). Start first with equation (B-44) and write it in terms relative to consumption of good 1, i.e.:

$$\frac{x_i}{x_1} = \frac{\alpha_i b_i u^{e_i b_i} \left( \frac{p_i}{Y} \right)^{b_i - 1}}{\alpha_1 b_1 u^{e_1 b_1} \left( \frac{p_1}{Y} \right)^{b_1 - 1}} \quad (\text{B-75})$$

This equation can be used to isolate  $\alpha_i$ :

$$\alpha_i = \frac{x_i}{x_1} \frac{\alpha_1 b_1 u^{e_1 b_1} \left(\frac{p_1}{Y}\right)^{b_1 - 1}}{b_i u^{e_i b_i} \left(\frac{p_i}{Y}\right)^{b_i - 1}} \quad (\text{B-76})$$

and then inserted into equation (B-44):

$$\sum_{i=1}^n \alpha_i u^{e_i b_i} \left(\frac{p_i}{Y}\right)^{b_i} = \alpha_1 u^{e_1 b_1} \frac{b_1}{s_1} \left(\frac{p_1}{Y}\right)^{b_1} \left[ \sum_{i=1}^n \frac{s_i}{b_i} \right] \equiv 1 \quad (\text{B-77})$$

The final expression in equation (B-77) can be used to solve for  $\alpha_1$  since the formula must equal 1 by definition:

$$\alpha_1 = u^{-e_1 b_1} \frac{s_1}{b_1} \left(\frac{Y}{p_1}\right)^{b_1} \left[ \sum_{i=1}^n \frac{s_i}{b_i} \right]^{-1} \quad (\text{B-78})$$

Substituting back into equation (B-77) we get:

$$\alpha_i = \frac{x_i}{b_i} u^{-e_i b_i} \left(\frac{Y}{p_i}\right)^{b_i - 1} \left[ \sum_{j=1}^n \frac{s_j}{b_j} \right]^{-1} \quad (\text{B-79})$$

The final calibration expression is then the following:

$$\alpha_i = \frac{s_i}{b_i} \left(\frac{Y}{p_i}\right)^{b_i} \frac{u^{-e_i b_i}}{\sum_{j=1}^n \frac{s_j}{b_j}} \quad (\text{B-80})$$

Utility is undefined in the base data and it is easiest to simply set it to 1.

In conclusion, for calibration we need the budget shares, initial prices, total expenditure, income elasticities and the own-price uncompensated elasticities. From this, we can derive base year consumption volumes, the Allen partial substitution elasticities through equation (B-69),  $\sigma$  (and therefore  $b$ ) through equation (B-72) and the inversion of the  $A$ -matrix,  $e$  through equation (B-74) and inversion of the  $B$ -matrix, and finally  $\alpha$  through equation (B-80).

It is possible that the initial shares and elasticities lead to inconsistent calibrated values for the  $b$  or  $e$  parameters. One solution, modified from Hertel (1997), is to implement some sort of maximum entropy method—explicitly imposing the constraints on the parameters. Step 1 is to calibrate the  $b$ -parameters using the following minimization problem:

$$\min L = \sum_i s_i (\varepsilon_{ii} - \varepsilon_{ii}^0)^2$$

subject to

$$\varepsilon_{ii} = (1 - b_i) (s_i - 1) - s_i \left[ b_i + \eta_i - \sum_j s_j b_j \right]$$

$$0 < b_i < 1$$

The loss function is a weighted some of square errors where  $\varepsilon^0$  represents the initial or target own-price elasticity and  $\varepsilon$  will be the estimated elasticity with the constraints holding. The first constraint is a transformation of equation (B-49) where the income elasticity is substituted into the definition of the own-price elasticity (swapping out for the yet unknown  $e$ -coefficients). One critical issue is to ascertain what income elasticities to use in the formula above. One could use the target income elasticities, or an initial transformation of the target elasticities such as described above.

The next step calibrates the  $e$ -parameters with some target income elasticities as given as well as the now calibrated  $b$ -parameters. The minimization problem is formulated as the following:

$$\min L = \sum_i s_i (\eta_i - \eta_i^0)^2$$

subject to

$$\eta_i = \frac{1}{\sum_k s_k e_k} \left[ e_i b_i - \sum_k s_k e_k b_k \right] - (b_i - 1) + \sum_k b_k s_k$$

$$\sum_i s_i \eta_i \equiv 1$$

$$(\eta_i - 1) (\eta_i^0 - 1) > 0$$

The final constraint insures that the estimated income elasticities preserve their relationship relative to 1, i.e. target elasticities lower than 1 remain lower than 1 in the estimation procedure.

### B.3.4 CDE in first differences

It is useful to decompose changes in demand using a linearized version of the demand function, and that which is used in the standard GEMPACK version of the CDE function. The CDE implicit utility function can be used to derive a relation between changes in income, utility and prices (all in per capita terms). The first step in the differentiation of the utility function, equation (B-42), leads to the following expression:

$$0 = \sum_i \alpha_i e_i b_i u^{e_i b_i - 1} \left( \frac{p_i}{Y} \right)^{b_i} du$$

$$- \sum_i \alpha_i b_i u^{e_i b_i} \left( \frac{p_i}{Y} \right)^{b_i - 1} \frac{p_i}{Y^2} dY$$

$$+ \sum_i \alpha_i b_i u^{e_i b_i} \left( \frac{p_i}{Y} \right)^{b_i - 1} \frac{1}{Y} dp_i$$

This can be simplified by inserting the expression for the demand equation, equation (B-44), and replacing demand with the budget shares ( $s_i$ ):

$$0 = \frac{du}{u} \sum_i e_i s_i - \frac{dY}{Y} \sum_i s_i + \sum_i s_i \frac{dp_i}{p_i}$$

And the final expression can be written as:

$$\dot{Y} = \sum_i e_i s_i \dot{u} + \sum_i s_i \dot{p}_i \tag{B-81}$$

where the dotted variables represent the percent change (and noting that the sum of the budget shares is equal to 1).

The differentiation of the demand function, equation (B-44) is somewhat more tedious. The first step leads to the following expression:

$$\begin{aligned}
 dx_i &= \alpha_i b_i e_i b_i u^{e_i b_i - 1} \left(\frac{p_i}{Y}\right)^{b_i - 1} \frac{du}{D} \\
 &+ \alpha_i b_i u^{e_i b_i} (b_i - 1) \left(\frac{p_i}{Y}\right)^{b_i - 2} \frac{1}{Y} \frac{dp_i}{D} \\
 &- \alpha_i b_i u^{e_i b_i} (b_i - 1) \left(\frac{p_i}{Y}\right)^{b_i - 2} \frac{p_i}{Y^2} \frac{dY}{D} \\
 &- \alpha_i b_i u^{e_i b_i} \left(\frac{p_i}{Y}\right)^{b_i - 1} D^{-2} \sum_j \alpha_j b_j e_j b_j u^{e_j b_j - 1} \left(\frac{p_j}{Y}\right)^{b_j} du \\
 &- \alpha_i b_i u^{e_i b_i} \left(\frac{p_i}{Y}\right)^{b_i - 1} D^{-2} \sum_j \alpha_j b_j b_j u^{e_j b_j} \left(\frac{p_j}{Y}\right)^{b_j - 1} \frac{1}{Y} dp_j \\
 &+ \alpha_i b_i u^{e_i b_i} \left(\frac{p_i}{Y}\right)^{b_i - 1} D^{-2} \sum_j \alpha_j b_j b_j u^{e_j b_j} \left(\frac{p_j}{Y}\right)^{b_j - 1} \frac{p_j}{Y^2} dY
 \end{aligned}$$

where  $D$  is the denominator in the demand equation. This can be simplified to the following expression in terms of the percent changes:

$$\begin{aligned}
 \dot{x}_i &= e_i b_i \dot{u} + (b_i - 1) \dot{p}_i - (b_i - 1) \dot{Y} \\
 &- \sum_j e_j b_j s_j \dot{u} - \sum_j b_j s_j \dot{p}_j + \sum_j b_j s_j \dot{Y}
 \end{aligned}$$

Re-grouping terms, the expression becomes:

$$\begin{aligned}
 \dot{x}_i &= (b_i - 1) \dot{p}_i - \sum_j b_j s_j \dot{p}_j \\
 &+ \dot{u} \left[ e_i b_i - \sum_j e_j b_j s_j \right] \\
 &+ \dot{Y} \left[ \sum_j b_j s_j - (b_i - 1) \right]
 \end{aligned}$$

The percent change in  $u$  can be replaced with the expression above, equation (B-81), to yield the following after re-arrangement:

$$\begin{aligned}
 \dot{x}_i &= (b_i - 1) \dot{p}_i - \sum_j b_j s_j \dot{p}_j - \frac{1}{\sum_k e_k s_k} \sum_j s_j \dot{p}_j \left[ e_i b_i - \sum_k e_k b_k s_k \right] \\
 &+ \dot{Y} \left[ \sum_k b_k s_k - (b_i - 1) + \frac{1}{\sum_k e_k s_k} \left( e_i b_i - \sum_k e_k b_k s_k \right) \right]
 \end{aligned}$$

The final formula inserts the formulas for the income and price elasticities from above to simplify further to the following expression:

$$\dot{x}_i = \sum_j \varepsilon_{ij} \dot{p}_j + \eta_i \dot{Y} \tag{B-82}$$

## Appendix C

# Analytical SAM and GTAP correspondence

This section describes the accounting framework used by the the model. Two different accounting frameworks will be described. The first describes the analytical SAM underlying the model, i.e. how the SAM can be constructed using the variables of the model. The second describes the links between the model variables and the initial database as provided by GTAP. The SAM is not a standard SAM and does not represent the full functionality of the model nor the underlying database. For example, demand is specified at the Armington level and the sales tax are collapsed to a single row dimension.

### C.1 The analytical SAM

The analytical SAM is the accounting framework that is derived from the variables of the model. In the absence of a shock, the analytical SAM should reproduce the input SAM. Table C.1 re-produces the analytical SAM. Below are a few explanatory notes:

1. The regional index ( $r$ ) is dropped for most of the expressions except when needed, for example in the bilateral trade flows.
2. The SAM reflects the differentiation between activities and commodities. The intersection of the activity rows with the commodity columns represents the transformation of domestic production into commodities. In the standard version of the model, the matrix is diagonal and there is a one-to-one mapping between activities and commodities.
3. The trade rows under the commodity columns represents the bilateral imports for region  $r$ . These are priced at border, i.e. CIF, prices. The commodity rows under the trade columns represent bilateral exports from region  $r$  and are valued at border, i.e. FOB, prices. At the global level, the difference between the two matrices represents the aggregate value of the trade and transport margins.
4. Outbound remittances are calculated post-tax. Inbound remittances accrue to households.
5. Taxes on profits are assumed to be net of depreciation. Outbound profits are calculated post-tax.



6. In the standard GTAP database, most direct taxes are levied on aggregate factor remuneration. There is no balancing between private and public savings. Direct household taxation is calibrated as a balancing item to clear the government accounts under the assumption that the deficit is zero. This can readily be modified with supplemental data. Direct household taxation can be negative, reflecting that net transfers between the government and households is positive.
7. The BOP row under the trade columns represents aggregate imports from each region (at border prices). Similarly, the trade rows under the BOP column represents the aggregate exports of region  $r$  towards each region. The BOP accounts reflect the full balance of payments. For each region, the sum of exports (including the export of trade and transport services) less the sum of imports, adjusted by remittances and net capital income, equals the net flow of foreign savings. At the global level, the foreign savings cell should be zero.

Table C.1: Analytical SAM

	<i>ACT(a)</i>	<i>COMM(i)</i>	<i>LAB(l)</i>
<i>Activities(a)</i>		$P_{a,i}X_{a,i}$	
<i>Commodities(i)</i>	$\gamma_{i,a}^{eda} PAT_i XA_{i,a}$		
<i>Labor(l)</i>	$W_{l,a}^p L_{l,a}^d$		
<i>Capital(cap)</i>	$\sum_v PK_{a,v}^p K_{a,v}^v$		
<i>Land(lnd)</i>	$PLand_a^p Land_a^d$		
<i>Nat. Res.(nrs)</i>	$PNRF_a^p XNRF_a^d$		
<i>Nat. Res.(wat)</i>	$PH2O_a^p H2O_a^d$		
<i>Indirect tax</i>	$\sum_i \tau_{i,a}^a \gamma_{i,a}^{eda} PAT_i XA_{i,a}$		
<i>Production tax</i>	$\tau_a^p PX_a XP_a$		
<i>Factor taxes</i>	$\sum_l \tau_{l,a}^l W_{l,a} L_{l,a}^d + \sum_v \tau_{a,v}^k PK_{a,v} K_{a,v}^v$ $+ \tau_a^t PLand_a Land_a^d + \tau_a^n PNRF_a XNRF_a^d$ $+ \tau_a^w PH2O_a H2O_a^d$		
<i>Import tax</i>		$\sum_s \tau_{s,i,r}^m PWM_{s,i,r} XW_{s,i,r}^d$	
<i>Export tax</i>		$\sum_d \tau_{r,i,d}^e PE_{r,i,d} XW_{r,i,d}^s$	
<i>Direct tax</i>			$\kappa_l^l \sum_a W_{l,a} L_{l,a}^d - \sum_d Remit_{d,l,r}$
<i>Household</i>			$(1 - \kappa_l^l) \sum_a W_{l,a} L_{l,a}^d$
<i>Government</i>			
<i>Investment</i>			
<i>Depreciation</i>			
<i>Trade margins</i>			
<i>Trade(s)</i>		$PWM_{s,i,r} XW_{s,i,r}^d$	
<i>Balance of payments</i>			$\sum_d Remit_{d,l,r}$

Table C.1 Analytical SAM, ctd.

	<i>CAP</i>	<i>LND</i>	<i>NRS</i>	<i>WAT</i>
<i>Activities(a)</i>				
<i>Commodities(i)</i>				
<i>Labor(l)</i>				
<i>Capital(cap)</i>				
<i>Land(lnd)</i>				
<i>Nat. Res.(nrs)</i>				
<i>Nat. Res.(wat)</i>				
<i>Indirect tax</i>				
<i>Production tax</i>				
<i>Factor taxes</i>				
<i>Import tax</i>				
<i>Export tax</i>				
<i>Direct tax</i>	$\kappa^k [\sum_a \sum_v PK_{a,v} K_{a,v}^v - DeprY]$	$\kappa^l \sum_a PLand_a Land_a^d$	$\kappa^n \sum_a PNRF_a XNRF_a^d$	$\kappa^w \sum_a PH2O_a H2O_a^d$
<i>Household</i>	$(1 - \kappa^k) [\sum_a \sum_v PK_{a,v} K_{a,v}^v - DeprY] - YQTF$	$(1 - \kappa^l) \sum_a PLand_a Land_a^d$	$(1 - \kappa^n) \sum_a PNRF_a XNRF_a^d$	$(1 - \kappa^w) \sum_a PH2O_a H2O_a^d$
<i>Government</i>				
<i>Investment</i>				
<i>Depreciation</i>	<i>DeprY</i>			
<i>Trade margins</i>				
<i>Trade(s)</i>				
<i>Balance of payments</i>	<i>YQTF<sub>τ</sub></i>			

Table C.1 Analytical SAM, ctd.

	<i>ITX</i>	<i>PTX</i>	<i>VTX</i>	<i>MTX</i>	<i>ETX</i>	<i>DTX</i>
<i>Activities(a)</i>						
<i>Commodities(i)</i>						
<i>Labor(l)</i>						
<i>Capital(cap)</i>						
<i>Land(lnd)</i>						
<i>Nat. Res.(nrs)</i>						
<i>Nat. Res.(wat)</i>						
<i>Indirect tax</i>						
<i>Production tax</i>						
<i>Factor taxes</i>						
<i>Import tax</i>						
<i>Export tax</i>						
<i>Direct tax</i>						
<i>Household</i>						
<i>Government</i>	$YGOV_{itx}$	$YGOV_{ptx}$	$YGOV_{vtx}$	$YGOV_{mtx}$	$YGOV_{etx}$	$YGOV_{dtx}$
<i>Investment</i>						
<i>Depreciation</i>						
<i>Trade margins</i>						
<i>Trade(s)</i>						
<i>Balance of payments</i>						

Table C.1 Analytical SAM, ctd.

	<i>HHD</i>	<i>GOV</i>	<i>INV</i>	<i>DEPR</i>
<i>Activities(a)</i>				
<i>Commodities(i)</i>	$\gamma_{i,h}^{eda} PAT_i^a XA_{i,h}$	$\gamma_{i,gov}^{eda} PAT_i^a XA_{i,gov}$	$\gamma_{i,inv}^{eda} PA_i^a XA_{i,inv}$	
<i>Labor(l)</i>				
<i>Capital(cap)</i>				
<i>Land(lnd)</i>				
<i>Nat. Res.(nrs)</i>				
<i>Nat. Res.(wat)</i>				
<i>Indirect tax</i>	$\sum_i \tau_{i,h}^a \gamma_{i,h}^{eda} PAT_i^a XA_{i,h}$	$\sum_i \tau_{i,gov}^a \gamma_{i,gov}^{eda} PAT_i^a XA_{i,gov}$	$\sum_i \tau_{i,inv}^a \gamma_{i,inv}^{eda} PAT_i^a XA_{i,inv}$	
<i>Production tax</i>				
<i>Factor taxes</i>				
<i>Import tax</i>				
<i>Export tax</i>				
<i>Direct tax</i>	$\kappa^h YH$			
<i>Household</i>				
<i>Government</i>				
<i>Investment</i>	$S^h$	$S^g$		<i>DeprY</i>
<i>Depreciation</i>				
<i>Trade margins</i>				
<i>Trade(s)</i>				
<i>Balance of payments</i>				

Table C.1 Analytical SAM, ctd.

	<i>TMG</i> ( <i>fd</i> )	<i>TRD</i> ( <i>d</i> )	<i>BOP</i>
<i>Activities</i> ( <i>a</i> )			
<i>Commodities</i> ( <i>i</i> )	$PDT_i XTT_i$	$\sum_d PWE_{r,i,d} XW_{r,i,d}$	
<i>Labor</i> ( <i>l</i> )			
<i>Capital</i> ( <i>cap</i> )			
<i>Land</i> ( <i>ln</i> <i>d</i> )			
<i>Nat. Res.</i> ( <i>nrs</i> )			
<i>Nat. Res.</i> ( <i>wat</i> )			
<i>Indirect tax</i>			
<i>Production tax</i>			
<i>Factor taxes</i>			
<i>Import tax</i>			
<i>Export tax</i>			
<i>Direct tax</i>			
<i>Household</i>			$\sum_s Remit_{r,l,s} + YQHT_r$
<i>Government</i>			
<i>Investment</i>			$PW^{sav} S_r^f$
<i>Depreciation</i>			
<i>Trade margins</i>			$\sum_i PDT_{r,i} XTT_{r,i}$
<i>Trade</i> ( <i>s</i> )			$\sum_d \sum_i PWE_{r,i,d} XW_{r,i,d}$
<i>Balance of payments</i>		$\sum_s \sum_i PWM_{s,i,r} XW_{s,i,r}$	

## C.2 Correspondence to GTAP database

Table C.2 provides the correspondence between the variables in the GAMS model and the standard GTAP database that is used as the starting basis for the GTAP model. Below are a few notes on the table:

1. The GTAP database reflects an Armington structure that is agent-specific (at the top nest). The model allows for either a national or agent-specific decomposition of domestic absorption across sources. The correspondence herein reflects national sourcing. Hence, the domestic/import split by agent in the GTAP database is aggregated to provide demand at the Armington level.
2. Subsidy costs in the GTAP database are entered as positive values. In the GAMS version of the model, subsidy rates are treated as negative tax rates and hence the costs will be negative. This affects the variables OSEP and FBEP. Note however that the variables OSEP and FBEP are ignored in the initialization routine and are the rates are calculated via other accounting identities.
3. The variable TVOM in the GTAP database is labeled as VOM by the aggregation facility.
4. The GTAP parameters VIFM, VDFM, VIFA and VDFFA contain the investment column. In GTAP, this is labeled as CGDS. These vectors are stored in a separate variable in the input database and are labeled VIIM, VDIM, VIIA and VDIA. In other words, they are treated like the two other final demand vectors—private and public consumption.
5. The GTAP database assumes a one-to-one correspondence between activities and commodities, which would be reflected into a one-to-one mapping between index  $i$  for commodities and index  $a$  for activities. The aggregation facility creates the diagonal 'make' matrix at the fully disaggregated level and uses separate mapping definitions for activities and commodities. This provides the user with full flexibility to create activities with joint production and commodities produced by more than one activity. The make matrix is stored under the name `TMAT`
6. GTAP's factor variables are treated in a single matrix. The concordance in the table reflects a break-out of the factors of production as in the model they are handled individually.

Table C.2: Correspondence between GAMS model variables and GTAP database

<i>GTAP</i>	<i>GAMS</i>	<i>Description</i>
VDFM+VIFM	$PAT_{r,i}XA_{r,i,a}$	Firms' (Armington) purchases at market prices
VDFA+VIDFA	$PA_{r,i,a}XA_{r,i,a}$	Firms' (Armington) purchases at market prices
VDFM+VIFM	$PAT_{r,i}XA_{r,i,h}$	Household (Armington) purchases at market prices
VDPA+VIPA	$PA_{r,i,h}XA_{r,i,h}$	Household (Armington) purchases at agents' prices
VDGM+VIGM	$PAT_{r,i}XA_{r,i,gov}$	Government (Armington) purchases purchases at market prices
VDGA + VIGA	$PA_{r,i,gov}XA_{r,i,gov}$	Government (Armington) purchases at agents' prices
VDFM(CGDS) + VIFM(CGDS)	$PAT_{r,i}XA_{r,i,inv}$	Investment (Armington) purchases at market prices
VDFA(CGDS) + VIFA(CGDS)	$PA_{r,i,inv}XA_{r,i,inv}$	Investment (Armington) purchases at agents' prices
VFM(1)	$W_{r,l,a}L_{r,l,a}^d$	Firms' labor purchases at market prices
VFM(cap)	$PK_{r,a}K_{r,a}^v$	Firms' capital purchases at market prices
VFM(1nd)	$PLand_{r,a}Land_{r,a}^d$	Firms' land purchases at market prices
VFM(nrs)	$PNRF_r XNRF_{r,a}^d$	Firms' nat. res. purchases at market prices
EVFA(1)	$W_{r,l,a}^a L_{r,l,a}^d$	Firms' labor purchases at agents' prices
EVFA(cap)	$PK_{r,a}^a K_{r,a}^v$	Firms' capital purchases at agents' prices
EVFA(1nd)	$PLand_{r,a}^a Land_{r,a}^d$	Firms' land purchases at agents' prices
EVFA(nrs)	$PNRF_r^a XNRF_{r,a}^d$	Firms' nat. res. purchases at agents' prices
TVOM	$PP_{r,a}XP_{r,a}$	Value of production at market (producer prices)
EVOA(1)	$(1 - \kappa_{r,l}^l) \sum_a W_{r,l,a} L_{r,l,a}^d$	After tax labor remuneration
EVOA(cap)	$(1 - \kappa_r^k) \sum_a PK_{r,a} K_{r,a}^v + \kappa_r^k DeprY$	After tax capital remuneration net of depreciation
EVOA(1nd)	$(1 - \kappa_r^t) \sum_a PLand_{r,a} Land_{r,a}^d$	After tax land remuneration
EVOA(nrs)	$(1 - \kappa_r^n) \sum_a PNRF_{r,a} XNRF_{r,a}^d$	After tax nat. res. remuneration
VKB	$K_r^s$	Initial capital stock
VDEP	$DeprY$	Value of depreciation
VIMS	$PDM_{s,i,r} XW_{s,i,r}^d$	Value of bilateral imports tariff inclusive
VIWS	$PWM_{s,i,r} XW_{s,i,r}^d$	Value of bilateral imports at border prices
VXWD	$PWE_{r,i,d} XW_{r,i,d}^s$	Value of bilateral exports at border prices
VXMD	$PE_{r,i,d} XW_{r,i,d}^s$	Value of bilateral exports at producer prices
VST	$PDT_{r,m} XTT_{r,m}$	Domestic exports of trade & transport services
VTWR	$PTMG_m XMG_{m,r,i,d}^m$	Value of margins from $r$ to $d$ for good $i$ using mode $m$
POP	$Pop_r$	Population—millions



# Appendix D

## User Guide

### D.1 Data preparation facility

#### D.1.1 Introduction

The data facility is composed of three modules.

- The first is the aggregation facility. It reads a user-prepared map file that contains the aggregation mappings. For example, a user may want to aggregate from GTAP's 140 regions and 57 sectors to some 25 regions and 10 sectors. The aggregation facility aggregates all relevant GTAP data files, the projection file for dynamic scenarios and the elasticities.
- The second module is called the 'filter' module. First developed by Tom Rutherford for his 'GTAPinGAMS' project<sup>1</sup> and integrated into Wolfgang Britz's CGEBox framework<sup>2</sup>, filtering is designed to remove small numeric values from the database that have minimal impact on data balances and model results. The 'Filter' program insures that the resulting database is nonetheless balanced.
- The third module is called the 'AlterTax' module. It is described in [Malcolm \(1998\)](#) and is a procedure to modify an initial GTAP database while minimizing changes to the structure of the original database. Though designed to change tax rates, it can be used to modify other features of a database. This module relies on the GTAP model itself, which is provided as part of the data preparation facility.

All three modules are designed to work with the 'geometry' of the standard GTAP database (albeit with the aggregated dimensions). In other words, the output of each of the three modules is identical in structure to the standard GTAP database. Users need to provide two mappings. One mapping is necessary for the data preparation facility, where the geometry is preserved. A second mapping is needed for the model aggregation where it is possible to move from the implicit diagonal make matrix of the standard GTAP geometry to a non-diagonal make matrix for the model. This allows for activities that can produce more than one commodity—say for instance all crop production is collapsed into a single activity, but crop demand is disaggregated. And it allows for a single commodity to be produced by more than one activity—for example electricity.

The outputs are contained in different containers. The output from the aggregation facility is found in the 'Agg' folder. The output of the filtering procedure is found in the 'Flt' folder.

---

<sup>1</sup> [Lanz and Rutherford \(2016\)](#)

<sup>2</sup> [http://www.ilr.uni-bonn.de/em/rsrch/cgebox/cgebox\\_GUI.pdf](http://www.ilr.uni-bonn.de/em/rsrch/cgebox/cgebox_GUI.pdf)

And the output of AlterTax is found in the 'Alt' folder. The end results, assuming all has worked appropriately, will be found in the 'Fnl' folder. Note that the filtering and AlterTax modules are optional.

Each data project is associated with a code name, for example '10x10'. This code name will be used for all output files and the aggregation facility will prepare a folder in the 'Data' directory that is named using the code name. All output folders will be contained in that directory.

### D.1.2 Preliminaries

There are no proscribed ways to setup the directory structures for the various components of Env10 so the setup below is just a suggestion. All of the components rely on the user providing full path names so it should be flexible enough to handle most user preferences. Figure 12 depicts a fairly standard layout. The root directory is *Env10* and it contains three obligatory folders: *Data*, *SatAcct* and *Model*. The first contains the routines for aggregating and preparing the data for the model. It also contains some sub-folders that will be further described below. The *SatAcct* contains miscellaneous data that is used by the aggregation facility and eventually the model. Use of most of the satellite data is optional and is user-specified. The *Model* folder contains the core code for the ENVISAGE Model. The remaining folders are user folders and will contain the files for single projects. Single projects can be identified with a specific aggregation, different dynamic assumptions and/or a different focus.

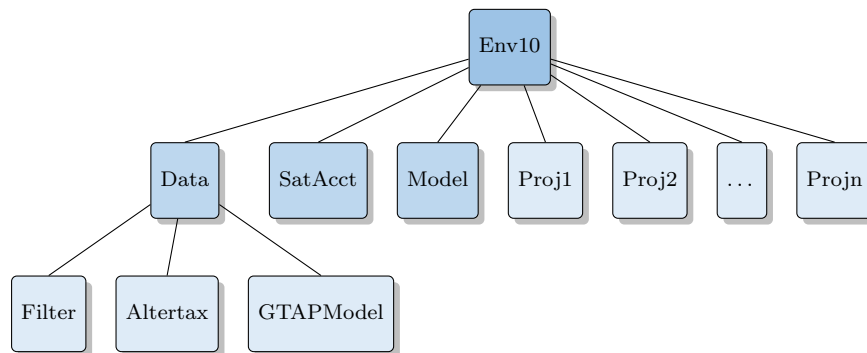


Figure 12: Possible directory structure

### Aggregation files

Table D.1 provides a list of files distributed with the data aggregation facility. In addition, the distribution will have a few sample 'MAP' files that users are likely to use as a starting point for new or modified aggregations. It is assumed that users will have access and appropriate licenses for the GTAP database files (see below for list of GTAP files). Most files need no user intervention. The exceptions would be the two base 'sets' definition files that would need to be updated with new database releases.

Table D.1: Files distributed with the aggregation facility

<i>File name</i>	<i>Description</i>
<b>Core code</b>	
AggGTAP.gms	Main aggregation program
aggSAM.gms	Code to output aggregate SAM in a CSV file
aggNRG.gms	Code to output aggregate energy and emissions in a CSV file
SSPSets.gms	Set definitions for reading SSP data file
saveMap.gms	Code to save aggregation mappings to be loaded into either Word/Excel or L <sup>A</sup> T <sub>E</sub> X
makeset.gms	Code to write out core set definitions for models, both GTAP and ENVISAGE
<b>Additional code for Envisage Model</b>	
makesetEnv.gms	Code to write additional sets for ENVISAGE
AggEnvElast.gms	Code to aggregate ENVISAGE elasticities
aggra.gms	Batinclude code for aggregating elasticities
aggrav.gms	Batinclude code for aggregating elasticities
aggrave.gms	Batinclude code for aggregating elasticities
makeAggSets.gms	Code to write final sets with appropriate suffixes
ConvertLabel.gms	Code to convert labels in aggregated parameter data file
<b>Base data set definitions</b>	
GTAPSets9_0.gms	Set definitions for standard GTAP database V9.0 (140 × 57)
GTAPSets9_0pw.gms	Set definitions for power+water GTAP database V9.0
GTAPSets9_2.gms	Set definitions for standard GTAP database V9.2 (141 × 57)
<b>Miscellaneous</b>	
makedata.cmd	Windows command file for facilitating all data processing
nipa.gms	Optional GAMS code that will output (in CSV format) the national income and product accounts for the base regions/countries in the GTAP database

The aggregation facility assumes that all of the GTAP-based data files have the same base name and are located in the same folder.<sup>3</sup> Note that some of the files downloaded from GAMS may not follow this convention, in which case they need to be renamed. All original GTAP files are delivered in the GEMPACK 'HAR' format. They need to be converted to GAMS' 'GDX' format for use by the aggregation facility. The program 'HAR2GDX.EXE' is a utility that will convert from 'HAR' format to 'GDX' format.<sup>4</sup>

Table D.2 provides a list of all possible data files that can be used with the data aggregation facility. Some are required, for obvious reasons. Others are optional and will depend on user-based options. Note that the power+water version of the GTAP database includes three extra headers compared to the standard GTAP database. It has total power generated by electricity in Gwhr, water withdrawals in irrigated crops and aggregate water withdrawals from other sectors of the economy.

<sup>3</sup> Referred to as '%GTAPBASE%' and '%gtpDir%', respectively, in the GAMS code.

<sup>4</sup> At times it can be the case that the header array in the HAR file does not correspond to what is expected in the GDX interface. The GTAP Center is working on ensuring consistency. One possible solution is to use the '/H' option when running HAR2GDX.

Table D.2: Base Data Files

<i>File name</i>	<i>Description</i>
<b>Required GTAP files</b>	
%GTAPBASE%DAT.gdx	File containing base SAM data
%GTAPBASE%PAR.gdx	File containing base parameter values for GTAP model
%GTAPBASE%VOLE.gdx	File containing base energy volume data
%GTAPBASE%EMISS.gdx	File containing base CO <sub>2</sub> emissions data
<b>Optional data files</b>	
%GTAPBASE%NCO2.gdx	File containing base non-CO <sub>2</sub> emissions data
%GTAPBASE%BOP.gdx	File containing remittances and cross-border profit flows. The former is sourced from the 'GMIG' database and the latter from the 'GDYN' database.
%GTAPBASE%ELAST.gdx	Data file containing key land and natural resource supply parameters. This file is not part of the standard GTAP suite and is described in more detail below [TBD]. If the file does not exist, these parameters will be set to default values.
%GTAPBASE%WAGES.gdx	File containing wage/employment splits for labor remuneration. This file is part of the 'GMIG' database. Its use is described below [TBD].
<b>Other data files</b>	
%SSPFILE%	Data file containing projections for population and GDP for 230 countries for all 5 Shared Socio-Economic Pathways (SSPs)—further described below [TBD]. In addition, the file contains the UN's population projections for the 2010, 2012 and 2015 revisions.
%GIDDLAB%	Data file containing an alternative wage/employment split. The World Bank's Global Income Distribution Dynamics (GIDD) Model provides an alternative set for employment levels. Note that the GIDD data is only provided for two types of labor—unskilled and skilled.
%GIDDPROJ%	Data file containing population projections (including education levels) from the GIDD.
<b>Parameter files</b>	
%EnvElast%	File containing disaggregated elasticities for use with the ENVISAGE Model. Two files are distributed. One for the standard GTAP database and the other for the power+water version.

### Filter module files

The filter module files are scattered in two folders—see Table D.3. The *Data* folder contains the calling `filter` routine and two user prepared files. The core files for the Filter module are all in the *Filter* sub-folder.

Table D.3: Files distributed with the Filter module

<i>File name</i>	<i>Description</i>
<b><i>Files in core 'Data' directory</i></b>	
Filter.gms	GAMS file invoking core filter code
<i>basename</i> Flt.gms	User prepared file with filter options
<i>basename</i> Map.gms	User prepared file with key mappings—same file used for Aggregation facility
<b><i>Additional code for Filter module in the 'Filter' directory</i></b>	
filter.gms	GAMS code containing core filter algorithm
remTinyValues.gms	Code to remove tiny value flows
itrlog.gms	Code that creates diagnostics for the Filter module
title.gms	Code mostly intended for the CGEBox GUI

### Altertax module files

The Altertax files are scattered in three folders, see Table D.4. The main *Data* folder contains `Altertax.gms` and the user options file `[basename]Alt.gms`. The sub-folder *GTAPModel* contains the core code for the GAMS-based GTAP model. The sub-folder *Altertax* contains a single file with the parameter definitions needed to run the GTAP model. Note that many of the parameters will be over-riden by Altertax to make all CES functions (and the utility function) Cobb-Douglas, further described below.

Table D.4: Files distributed with the Altertax module

<i>File name</i>	<i>Description</i>
<b><i>Files in core 'Data' directory</i></b>	
Altertax.gms	GAMS file invoking core GTAP model code
<i>basename</i> Alt.gms	User prepared file with the Altertax shock
<b><i>Additional code for Altertax module in the 'GTAPModel' directory</i></b>	
model.gms	GAMS code containing core GTAP model specification
cal.gms	GAMS code containing variable and parameter initialization and calibration
iterloop.gms	Code containing instructions to prepare a new solve statement
solve.gms	Code invoking a GAMS solver
postsim.gms	Code saving model results in a CSV-formatted file
emiCSV.gms	Code to save emission results in a CSV-formatted file
saveData.gms	Code saving model results in GDX-formatted files with the same structure as the input GTAP-based data files
<b><i>Additional code for Altertax module in the 'AlterTax' directory</i></b>	
AlterTaxPrm.gms	GAMS code containing default elasticities for GTAP model.

### Data sub-folders

Each aggregation project, with its code, will be associated with a data folder that has the same name as the project. So if the code for the project is '10x10', for example, the data from the

data preparation modules will be contained in the folder '10x10'. The output from each module is contained in its own sub-folder of which there are four named Agg, Flt, Alt and Fnl. The last folder contains the final results from running the three modules. If a module is skipped, all the data files from the previous module should be copied over to the next folder. Users can create these folders, or use the automated Windows command file called `makedata.cmd` that is further described below. The `makeData` command file also automates the moving of files across folders. Figure 13 provides a visual guide to the directory structure of the output data.

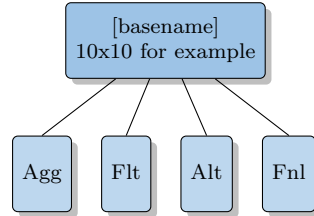


Figure 13: Directory structure of output data

### D.1.3 Aggregation

From the user perspective aggregation involves the preparation of the map file that will contain the aggregation mappings as well as miscellaneous other information required for the data facility. The mapping file has two components. The first part relates to a generic aggregation of the GTAP database and its components. It hones closely to the GTAPAgg and FlexAgg facilities available for GEMPACK users. The second component is specific to aggregation for the ENVISAGE Model.

#### Global options

The first part of a map file contains global options for the aggregation. Listing D.1 provides an example of the preamble statements for a listing file. The first line is an include file for the base data's set definitions. The distribution comes with two prepared set definition files—one for the standard GTAP database and the other for the power+water database. The following set of lines provide information for locating the input databases. The first is the location of all of the GTAP data files (DAT, PAR, VOLE, EMISS, NCO2, BOP, ELAST and WAGES). The second line provides the base name for the GTAP files, for example GSD or GSDPW—the latter stands for the power+water database. The remaining four lines refer to the full path name of the other input databases—the SSP projection file, the two GIDD databases and the file of disaggregated ENVISAGE elasticities.

Listing D.1: Global options for the MAP file

```

1  $include "GTAPsets9.0.gms"

3  $setGlobal gtpDir      "V:\GTAP9\V9RC2\"
4  $setGlobal GTAPBASE   "GSD"
5  $setGlobal SSPFile    "V:\ClimateChange\AR5\SSP\sspScenV9\sspScenV9.gdx"
6  $setGlobal giddLab    "..\SatAcct\giddLab.gdx"
7  $setGlobal giddProj   "..\SatAcct\giddProj.gdx"
8  $setGlobal EnvElast   "V:\GTAP9\Elast\EnvLinkElast.gdx"

10 $setGlobal DYN        ON
11 $setGlobal ifPower    OFF
12 $setGlobal ifWater    OFF
13 $setGlobal NCO2       ON
14 $setGlobal ELAST      OFF
  
```

```
15 $setGlobal LAB          ON
16 $setGlobal BoP         ON
17 $setGlobal SAVEMAP     TXT

19 * Only used to override GTAP parameters for Env model

21 $setGlobal OVRRIDEGTAPARM 0
22 $setGlobal OVRRIDEGTAPINC 0

24 * Select a labor option
25 * Valid options are:
26 *   noLab — ignore employment volumes (all wages are set to 1)
27 *   agLab — calculate ag and non-ag employment (wages uniform within zones)
28 *   allLab — assume employment data is correct for each sector (wages differ for each sector)
29 *   giddLab — Use the GIDD labor data

31 $macro IFLABOR noLab
```

---

The remaining set of options turn on or off various features of the aggregation. These are described in further detail in Table D.5.

Table D.5: Global options for the MAP file

<i>File name</i>	<i>Description</i>
DYN	If this option is set to ON, the aggregation facility will read the SSP and GIDD projection files and create the scenario file with the appropriate aggregation. This is to be used for dynamic versions of the model.
ifPower	If this option is set to ON, the aggregation facility will retrieve the Gwhr information from the GTAP (power) database and aggregate.
ifWater	If this option is set to ON, the aggregation facility will retrieve the water volume information from the GTAP (water) database and aggregate.
NCO2	If this option is set to ON, the aggregation facility will retrieve the non-CO <sub>2</sub> information from the non-CO <sub>2</sub> database and aggregate.
ELAST	If this option is set to ON, the aggregation facility will retrieve the land and natural resource parameters from the relevant 'ELAST' file. It should be set to OFF for an ENVISAGE aggregation as the same parameters are contained in the parameter file.
LAB	If this option is set to ON, the aggregation facility will retrieve the employment data from one of the available databases—either GTAP or the GIDD—and aggregate. The exact type of aggregation will depend on the IFLABOR option described below.
BOP	If this option is set to ON, the aggregation facility will retrieve the additional balance of payments data from the 'BOP' database and aggregate.
SAVEMAP	This option takes two values: 'TXT' and 'LATEX'. The 'TXT' option will save the aggregation mappings in a file called '[baseBame]Sets.txt'. It is a comma delimited text file (CSV) that contains the aggregation mappings from the original GTAP database to the model-level aggregation. It can be loaded into an Excel file and the <code>Data Convert to Columns</code> command can be used to convert the data into a table with three columns, using the 'comma' delimiter option. It is then easy to copy the table into Word. The 'LATEX' option creates a file with the name '[baseName]Sets.tex' and creates three relatively generic tables that can be included in a L <sup>A</sup> T <sub>E</sub> X file. The user may have to edit the file to convert special characters—such as underscores ('_') and ampersands ('&').
OVERRIDEGTAPARM	Currently ignored and to be reviewed.
OVERRIDEGTAPINC	If this option is set to 1, the income elasticities will be taken from the input base parameter file. If the option is set to 0, the GTAP income elasticities prevail and override the income elasticities from the parameter file.
IFLABOR	This option takes four values. The first value is 'noLab'. In this case the employment volumes are ignored and in effect all wages are set to 1. The second value is 'agLab'. In this case the volume data is used to aggregate agricultural and non-agricultural employment. Within each of these two broad sectors wages are assumed to be uniform across sectors and in effect the implicit inter-sectoral wage differences are ignored. The purpose of this is to minimize potential welfare implications by assuming heterogeneous inter-sectoral wages (apart from the ag/non-ag distinction). The third option is 'allLab' in which case the employment volume data is fully utilized and explicit inter-sectoral wages are assumed to hold. The fourth option is 'giddLab' in which case the GIDD employment volumes are used to determine the wage/employment split. Note that the GIDD option implies a skill/non-skilled labor aggregation.



## Standard mappings

The second part of the mapping file contains the standard mappings—this is aimed for the GTAP model and hones very closely to GTAPAGG, the GEMPACK-based aggregation facility. Users define the aggregated sets for activities/commodities, regions and factors of production—with their respective mappings to the full GTAP database. Users have some flexibility in how to define these sets and mappings and the distribution comes with two examples.

Listing D.2 provides an example that is virtually identical to the 10x10 mapping example from GTAP. A first set *act* is defined that contains the 10 aggregate sectors. The set *a* is the union of *act* with the *cgds* activity that represents the capital goods sector (or investment expenditures). The set *i* on the other hand is defined to be the same as *act*. With the exception of *cgds* there is a one-to-one mapping between the sets *a* and *i*, i.e. the implicit assumption of a diagonal make matrix. Similarly, the user defines set *r* for regions and *fp* for the factors of production.

These basic sets are subsequently decomposed into a number of subsets—most of which are fairly obvious. Some do have implications for the model:

- The subset *ul* is described as the set of unskilled labor. In fact, it is the set of labor types that substitute for the capital/skilled bundle.<sup>5</sup> It is possible to assign all labor types to the subset *ul*, in which case the so-called skilled labor bundle will be empty. The subset *sl* will be determined as the complement of the subset *ul*.
- The subset *lr* determines the labor types used to calculate the skill premium. This became necessary with the new GTAP database that has 5 labor types.
- The *wat* subset should be empty in the absence of the water factor—only available with the power+water database.
- The *mapz* mapping defines the ag vs. non-ag activities. This is only used to determine ag vs. non-ag wages in the case the 'IFLABOR' option is set to *agLab*.
- The set *mapt* will instruct the aggregation facility to merge land and capital payments into capital payments only—for the selected activities. For example, it is possible to remove the land payment in other livestock (the *oap* sector).
- The set *mapn* has the same purpose, but for the natural resource factor. For example, one may wish to delete the natural resource factor payment if the *omn* activity is merged with manufacturing; as depending on the supply elasticity of the natural resource, this may overly restrict output expansion (or contraction).

Post-simulation analysis of the results allow for aggregation of activities, commodities, regions and labor. These are at the full discretion of the user. The aggregate sets are given respectively by *aga*, *ia*, *ra* and *lagg*. The aggregation facility will automatically append the suffixes '-a' and '-c' for the individual activities and commodities, respectively. So it would be usual practice to define the aggregation set items with the '-a' and '-c' suffixes for compatibility. For purposes of post-simulation, these sets will also include the individual set items. For example, the aggregation facility will write out the final set *ra* as the union of the set *r* and *ra*, but the user only needs to provide the definitions for the aggregate items as the aggregation facility will take care of merging the two sets. The definition of these aggregations (with their respective mappings) simplifies the output code. For example, the code to aggregate GDP across regions takes the form:

<sup>5</sup> This determines which labor types are assigned to the *LAB1* and *LAB2* bundles.

---

```
2 loop(ra, put ra.tl, (sum(r$mapra(ra,r), gdp(r)) / ; ) ;
```

---

where the mapping *mapra* defines the aggregation mapping for each region *ra*, including the individually modeled regions.

The aggregation facility requires the factor mobility parameters for the GTAP model. This is given in the parameter *etrae1*.

The aggregation mappings are relatively self-explanatory. The exception may be the mapping *mapl* that will match the user-input labor types to the GIDD labor labels that are fixed.

Listing D.2: Standard mappings

---

```
1 sets
3 act "Base activities" /
4 GrainsCrops "Grains and Crops"
5 MeatLstk "Livestock and Meat Products"
6 Extraction "Mining and Extraction"
7 ProcFood "Processed Food"
8 TextWapp "Textiles and Clothing"
9 LightMnfc "Light Manufacturing"
10 HeavyMnfc "Heavy Manufacturing"
11 Util_Cons "Utilities and Construction"
12 TransComm "Transport and Communication"
13 OthServices "Other Services"
14 /
16 a "All activities" /
17 set.act
18 cgds "Capital goods"
19 /
21 i(a) "Commodities" /
22 set.act
23 /
25 r "Regions" /
26 Oceania "Australia, New Zealand"
27 EastAsia "East Asia"
28 SEAsia "Southeast Asia"
29 SouthAsia "South Asia"
30 NAmerica "North America"
31 LatinAmer "Latin America"
32 EU_25 "European Union 25"
33 MENA "Middle East and North Africa"
34 SSA "Sub-Saharan Africa"
35 RestofWorld "Rest of World"
36 /
38 fp "Factors of production" /
39 UnSkLab "Unskilled labor"
40 SkLab "Skilled labor"
41 Capital "Capital"
42 Land "Land"
43 NatRes "Natural resource"
44 /
46 l(fp) "Labor factors" /
47 UnSkLab "Unskilled labor"
48 SkLab "Skilled labor"
49 /
50 ul(l) "Unskilled labor" /
51 UnSkLab "Unskilled labor"
52 SkLab "Skilled labor"
53 /
```

```

54 lr(l) "Reference labor for skill premium" /
55 SkLab      "Skilled labor"
56 /
57 cap(fp) "Capital" /
58 Capital    "Capital"
59 /
60 lnd(fp) "Land endowment" /
61 Land       "Land"
62 /
63 nrs(fp) "Natural resource" /
64 NatRes     "Natural resource"
65 /
66 wat(fp) "Water resource" /
67 /

69 ra "Aggregate regions for emission regimes and model output" /
70 hic "High-income countries"
71 lmy "Developing countries"
72 wld "World Total"
73 /
74 ia "Aggregate commodities for model output" /
75 tagr-c "Agriculture"
76 tman-c "Manufacturing"
77 tsrv-c "Services"
78 toth-c "Other"
79 ttot-c "Total"
80 /
81 aga "Aggregate activities for model output" /
82 tagr-a "Agriculture"
83 tman-a "Manufacturing"
84 tsrv-a "Services"
85 toth-a "Other"
86 ttot-a "Total"
87 /
88 lagg "Aggregate labor for model output" /
89 tot  "Total labor"
90 /
91 ;

93 set cgds(a) / cgds / ;

95 * User defined parameters (i.e. not aggregated by aggregation facility

97 parameter etrael(fp) "CET transformation elasticities for factor allocation" /
98 UnSkLab  inf
99 SkLab    inf
100 Capital  inf
101 Land     1.0
102 NatRes   0.001
103 / ;

105 * This zonal mapping is for labor/volume splits between agriculture and other

107 set mapz(z,a) "Mapping of activities to zones" /
108 rur.(GrainsCrops,MeatLstk)
109 / ;

111 mapz("urb",i) = not mapz("rur",i) ;
112 mapz("nsg",i) = yes ;

114 * >>>> MUST INSERT RESIDUAL REGION (ONLY ONE)

116 set rres(r) "Residual region" /

118 NAmerica

120 / ;

```

```
122 * >>>> MUST INSERT MUV REGIONS (ONE OR MORE)

124 set rmuv(r) "MUV regions" /

126 Oceania
127 EastAsia
128 NAmerica
129 EU_25

131 / ;

133 set mapt(a) "Merge land and capital payments in the following sectors" /

135 / ;

137 set mapn(a) "Merge natl. res. and capital payments in the following sectors" /

139 / ;

141 * MAPPINGS TO GTAP

143 set mapa(prod.comm,a) /
144 pdr . GrainsCrops
145 wht . GrainsCrops
146 gro . GrainsCrops
147 v_f . GrainsCrops
148 osd . GrainsCrops
149 c_b . GrainsCrops
150 pfb . GrainsCrops
151 ocr . GrainsCrops
152 ctl . MeatLstk
153 oap . MeatLstk
154 rmk . MeatLstk
155 wol . MeatLstk
156 frs . Extraction
157 fsh . Extraction
158 coa . Extraction
159 oil . Extraction
160 gas . Extraction
161 omn . Extraction
162 cmt . MeatLstk
163 omt . MeatLstk
164 vol . ProcFood
165 mil . ProcFood
166 pcr . GrainsCrops
167 sgr . ProcFood
168 ofd . ProcFood
169 b_t . ProcFood
170 tex . TextWapp
171 wap . TextWapp
172 lea . LightMnfc
173 lum . LightMnfc
174 ppp . LightMnfc
175 p_c . HeavyMnfc
176 crp . HeavyMnfc
177 nmm . HeavyMnfc
178 i_s . HeavyMnfc
179 nfm . HeavyMnfc
180 fmp . LightMnfc
181 mvh . LightMnfc
182 otn . LightMnfc
183 ele . HeavyMnfc
184 ome . HeavyMnfc
185 omf . LightMnfc
186 ely . Util.Cons
187 gdt . Util.Cons
188 wtr . Util.Cons
189 cns . Util.Cons
```

```
190 trd . TransComm
191 otp . TransComm
192 wtp . TransComm
193 atp . TransComm
194 cmn . TransComm
195 ofi . OthServices
196 isr . OthServices
197 obs . OthServices
198 ros . OthServices
199 osg . OthServices
200 dwe . OthServices
201 / ;

203 mapa(prod.comm, cgds)$ (sameas("CGDS", prod.comm)) = yes ;

205 set mapr(reg,r) /
206 aus . Oceania
207 nzl . Oceania
208 xoc . Oceania
209 chn . EastAsia
210 hkg . EastAsia
211 jpn . EastAsia
212 kor . EastAsia
213 mng . EastAsia
214 twn . EastAsia
215 xea . EastAsia
216 brn . EastAsia
217 khm . SEAsia
218 idn . SEAsia
219 lao . SEAsia
220 mys . SEAsia
221 phl . SEAsia
222 sgp . SEAsia
223 tha . SEAsia
224 vnm . SEAsia
225 xse . SEAsia
226 bgd . SouthAsia
227 ind . SouthAsia
228 npl . SouthAsia
229 pak . SouthAsia
230 lka . SouthAsia
231 xsa . SouthAsia
232 can . NAmerica
233 usa . NAmerica
234 mex . NAmerica
235 xna . NAmerica
236 arg . LatinAmer
237 bol . LatinAmer
238 bra . LatinAmer
239 chl . LatinAmer
240 col . LatinAmer
241 ecu . LatinAmer
242 pry . LatinAmer
243 per . LatinAmer
244 ury . LatinAmer
245 ven . LatinAmer
246 xsm . LatinAmer
247 cri . LatinAmer
248 gtm . LatinAmer
249 hnd . LatinAmer
250 nic . LatinAmer
251 pan . LatinAmer
252 slv . LatinAmer
253 xca . LatinAmer
254 dom . LatinAmer
255 jam . LatinAmer
256 pri . LatinAmer
257 tto . LatinAmer
```

---

258 xcb . LatinAmer  
259 aut . EU\_25  
260 bel . EU\_25  
261 cyp . EU\_25  
262 cze . EU\_25  
263 dnk . EU\_25  
264 est . EU\_25  
265 fin . EU\_25  
266 fra . EU\_25  
267 deu . EU\_25  
268 grc . EU\_25  
269 hun . EU\_25  
270 irl . EU\_25  
271 ita . EU\_25  
272 lva . EU\_25  
273 ltu . EU\_25  
274 lux . EU\_25  
275 mlt . EU\_25  
276 nld . EU\_25  
277 pol . EU\_25  
278 prt . EU\_25  
279 svk . EU\_25  
280 svn . EU\_25  
281 esp . EU\_25  
282 swe . EU\_25  
283 gbr . EU\_25  
284 che . RestofWorld  
285 nor . RestofWorld  
286 xef . RestofWorld  
287 alb . RestofWorld  
288 bgr . RestofWorld  
289 blr . RestofWorld  
290 hrv . RestofWorld  
291 rou . RestofWorld  
292 rus . RestofWorld  
293 ukr . RestofWorld  
294 xee . RestofWorld  
295 xer . RestofWorld  
296 kaz . RestofWorld  
297 kgz . RestofWorld  
298 xsu . RestofWorld  
299 arm . RestofWorld  
300 aze . RestofWorld  
301 geo . RestofWorld  
302 bhr . MENA  
303 irn . MENA  
304 isr . MENA  
305 jor . MENA  
306 kwt . MENA  
307 omn . MENA  
308 qat . MENA  
309 sau . MENA  
310 tur . MENA  
311 are . MENA  
312 xws . MENA  
313 egy . MENA  
314 mar . MENA  
315 tun . MENA  
316 xnf . MENA  
317 ben . SSA  
318 bfa . SSA  
319 cmr . SSA  
320 civ . SSA  
321 gha . SSA  
322 gin . SSA  
323 nga . SSA  
324 sen . SSA  
325 tgo . SSA

---

```

326 xwf . SSA
327 xcf . SSA
328 xac . SSA
329 eth . SSA
330 ken . SSA
331 mdg . SSA
332 mwi . SSA
333 mus . SSA
334 moz . SSA
335 rwa . SSA
336 tza . SSA
337 uga . SSA
338 zmb . SSA
339 zwe . SSA
340 xec . SSA
341 bwa . SSA
342 nam . SSA
343 zaf . SSA
344 xsc . SSA
345 xtw . RestofWorld
346 / ;

348 set mapf(endw_comm, fp) /
349 ag_othlowsk . UnSkLab
350 service_shop . UnSkLab
351 clerks . UnSkLab
352 tech_aspros . SkLab
353 off_mgr_pros . SkLab
354 Capital . Capital
355 Land . Land
356 NatlRes . NatRes
357 / ;

359 set mapl(lg, 1) "Mapping to GIDD labor database" /
360 nsk.UnSkLab
361 skl.SkLab
362 / ;

```

---

## Sets and mappings for model aggregation

As noted above, the model aggregation can differ from the data aggregation. The data aggregation is built around the concept of a diagonal make matrix—this ensures that the output from any of the three modules always has the same ‘geometry’. The model aggregation is intended to allow for a non-diagonal make matrix—this can vastly simplify a model, for example in incorporating the recent GTAP power database. This section allows the user to input the sets and mappings for the model aggregation. Listing D.3 has an example from the ‘10x10’ map file.

Most of the entries in the listing file should be fairly self-explanatory. The user defines two sets, `actf` and `commf`, that will represent the model aggregation. They can replicate the set  $i$  from the first section, in which case the make matrix will be the standard diagonal. In the example from the listing file, the two agricultural activities from the data aggregation are merged into one activity called **Agriculture**. Thus the single activity **Agriculture** will produce two commodities: **GrainsCrops** and **MeatLstk**. This is reflected in the two mapping sets `mapaf` and `mapif`. Note that the mappings are based on the data aggregation and not the original GTAP database.<sup>6</sup>

The model needs an `imuvf` subset based on the final model aggregation. This is followed by the definition of the aggregations of commodities, activities and regions, i.e. the set mappings `mapia`,

---

<sup>6</sup> The aggregation facility will automatically append the ‘-a’ and ‘-c’ suffixes to activities and commodities, respectively, except for the user-defined aggregations.

mapaga and mapra. Note that these mappings are ignored by the model and typically are only used for post-processing of results.

Listing D.3: Sets for model aggregation

```

1 * -----
2 *
3 *   Section dealing with model aggregations (to handle non-diagonal make matrix)
4 *
5 * -----
7 *   Model aggregation(s)
9 set actf "Model activities" /
10     Agriculture      "Agriculture"
11     Extraction       "Mining and Extraction"
12     ProcFood         "Processed Food"
13     TextWapp         "Textiles and Clothing"
14     LightMnfc        "Light Manufacturing"
15     HeavyMnfc        "Heavy Manufacturing"
16     Util.Cons        "Utilities and Construction"
17     TransComm        "Transport and Communication"
18     OthServices      "Other Services"
19 / ;
21 set commf "Model commodities" /
22     GrainsCrops      "Grains and Crops"
23     MeatLstk         "Livestock and Meat Products"
24     Extraction       "Mining and Extraction"
25     ProcFood         "Processed Food"
26     TextWapp         "Textiles and Clothing"
27     LightMnfc        "Light Manufacturing"
28     HeavyMnfc        "Heavy Manufacturing"
29     Util.Cons        "Utilities and Construction"
30     TransComm        "Transport and Communication"
31     OthServices      "Other Services"
32 / ;
34 set mapaf(i, actf) "Mapping from original to modeled activities" /
35     GrainsCrops      .Agriculture
36     MeatLstk         .Agriculture
37     Extraction       .Extraction
38     ProcFood         .ProcFood
39     TextWapp         .TextWapp
40     LightMnfc        .LightMnfc
41     HeavyMnfc        .HeavyMnfc
42     Util.Cons        .Util.Cons
43     TransComm        .TransComm
44     OthServices      .OthServices
45 / ;
47 set mapif(i, commf) "Mapping from original to modeled commodities" /
48     GrainsCrops      .GrainsCrops
49     MeatLstk         .MeatLstk
50     Extraction       .Extraction
51     ProcFood         .ProcFood
52     TextWapp         .TextWapp
53     LightMnfc        .LightMnfc
54     HeavyMnfc        .HeavyMnfc
55     Util.Cons        .Util.Cons
56     TransComm        .TransComm
57     OthServices      .OthServices
58 / ;
60 * >>>> MUST INSERT MUV COMMODITIES (ONE OR MORE)
61 *     !!!! Be careful of compatibility with modeled imuv
62 *     This one is intended for AlterTax

```



```

64 set imuvf(commf) "MUV commodities" /
66 ProcFood
67 TextWapp
68 LightMnfc
69 HeavyMnfc
71 / ;
73 * >>>> Aggregation of modeled sectors and regions
75 set mapia(ia,commf) "mapping of individual comm to aggregate comm" /
76 tagr-c.GrainsCrops
77 tagr-c.MeatLstk
78 tman-c.ProcFood
79 tman-c.TextWapp
80 tman-c.LightMnfc
81 tman-c.HeavyMnfc
82 toth-c.Extraction
83 tsrv-c.Util_Cons
84 tsrv-c.TransComm
85 tsrv-c.OthServices
86 / ;
87 mapia("ttot-c",commf) = yes ;
89 set mapaga(aga,actf) "mapping of individual comm to aggregate comm" /
90 tagr-a.Agriculture
91 tman-a.ProcFood
92 tman-a.TextWapp
93 tman-a.LightMnfc
94 tman-a.HeavyMnfc
95 toth-a.Extraction
96 tsrv-a.Util_Cons
97 tsrv-a.TransComm
98 tsrv-a.OthServices
99 / ;
100 mapaga("ttot-a",actf) = yes ;
102 set mapra(ra,r) "Mapping of model regions to aggregate regions" /
103 hic.(Oceania, NAmerica, EU.25)
104 / ;
105 mapra("lmy", r)$(not mapra("hic",r)) = yes ;
106 mapra("wld", r) = yes ;
108 set maplagg(lagg,l) "Mapping of model labor to aggregate labor" ;
109 maplagg("Tot",l) = yes ;

```

## Sorted sets

GAMS does not preserve the ordering of items in a set if the items have appeared in a previous set definition. To facilitate the loading of model results in Excel, the user can enter a desired sort for regions, activities and commodities. Listing D.4 is an example of the sort mappings for the '10x10' aggregation. The user needs to provide three separate sort mappings: one for regions, one for activities and one for commodities. The aggregation facility will use this information to create a sort mapping for model output use—including a sort mapping for the SAM labels.

Listing D.4: Mapping for sorted sets

```

1 set sortOrder / sort1*sort500 / ;
2 set mapRegSort(sortOrder,r) /
4 sort1 . Oceania
5 sort2 . EastAsia

```

---

```

6   sort3 . SEAsia
7   sort4 . SouthAsia
8   sort5 . NAmerica
9   sort6 . LatinAmer
10  sort7 . EU.25
11  sort8 . MENA
12  sort9 . SSA
13  sort10. RestofWorld

15 / ;

17 set mapActSort (sortOrder,actf) /

19  sort1 .Agriculture
20  sort2 .Extraction
21  sort3 .ProcFood
22  sort4 .TextWapp
23  sort5 .LightMnfc
24  sort6 .HeavyMnfc
25  sort7 .Util.Cons
26  sort8 .TransComm
27  sort9 .OthServices

29 / ;

31 set mapCommSort (sortOrder,commf) /
32  sort1 .GrainsCrops
33  sort2 .MeatLstk
34  sort3 .Extraction
35  sort4 .ProcFood
36  sort5 .TextWapp
37  sort6 .LightMnfc
38  sort7 .HeavyMnfc
39  sort8 .Util.Cons
40  sort9 .TransComm
41  sort10 .OthServices
42 / ;

```

---

## Sets and mappings for the Envisage Model

The final part of the map file is specific to the ENVISAGE Model. Many are critical components of ENVISAGE as they often determine model specification. For example, the first two subsets, **acr** and **alv** determine which activities use the crop and livestock production structure, respectively. All other activities will be assigned to the subset **ax** that is the default production structure. Listing D.5 provides an example for the 10x10 map file, though we will provide additional snippets below for a different aggregation tuned to the power and water modules of the model.

The subsets **agr** and **man** are not formally part of the model. These subsets are currently being used to determine activity-specific productivity shifters in dynamic scenarios.<sup>7</sup> They could potentially be used as well in post-simulation processing. The subsets **aenergy**, **affl** and **aw** are currently not being used.

Listing D.5: Sets for the ENVISAGE Model aggregation

---

```

1 * -----
2 *
3 *   Envisage section
4 *
5 * -----

```

---

<sup>7</sup> See the model file `initScen.gms`

```

7 * >>>> Activity related sets and subsets

9 set acr(actf)  "Crop activities" /
10 / ;

12 set alv(actf)  "Livestock activities" /
13 / ;

15 set agr(actf)  "Agricultural activities" /
16     Agriculture  "Agriculture"
17 / ;

19 set man(actf)  "Manufacturing activities" /
20     Extraction   "Mining and Extraction"
21     ProcFood     "Processed Food"
22     TextWapp     "Textiles and Clothing"
23     LightMnfc    "Light Manufacturing"
24     HeavyMnfc    "Heavy Manufacturing"
25 / ;

27 set aenergy(actf) "Energy activities" /
28     Extraction   "Mining and Extraction"
29     Util.Cons    "Utilities and Construction"
30 / ;

32 set affl(actf)  "Fossil fuel activities" /
33     Extraction   "Mining and Extraction"
34 / ;

36 set aw(actf)    "Water services activities" /
37 / ;

39 set elya(actf)  "Power activities" /
40 / ;

42 set etd(actf)  "Electricity transmission and distribution activities" /
43 / ;

45 set pb         "Power bundles" /
46     othp       "Other power"
47 / ;

49 set mappow(pb,elya) "Mapping of power activities to power bundles" /
50 / ;

52 * >>>> Commodity sets and subsets

54 set frt(commf)  "Fertilizer commodities" /
55 / ;

57 set feed(commf) "Feed commodities" /
58 / ;

60 set iw(commf)   "Water services commodities" /
61 / ;

63 set e(commf)    "Energy commodities" /
64 $ontext
65     Extraction   "Mining and Extraction"
66     Util.Cons    "Utilities and Construction"
67 $offtext
68 / ;

70 set elyc(commf) "Electricity commodities" /
71     Util.Cons    "Utilities and Construction"
72 / ;

74 set f(commf)    "Fuel commodities" /

```

```

75 *      Extraction      "Mining and Extraction"
76 / ;

78 * This zonal mapping is for labor market segmentation in final model

80 set mapzf(z,actf)  "Mapping of activities to zones" /
81   rur.Agriculture
82 / ;

84 mapzf("urb",actf) = not mapzf("rur",actf) ;
85 mapzf("nsg",actf) = yes ;

87 * >>>> Household commodity section

89 set k "Household commodities" /
90   GrainsCrops      "Grains and Crops"
91   MeatLstk         "Livestock and Meat Products"
92   ProcFood         "Processed Food"
93   TextWapp         "Textiles and Clothing"
94   LightMnfc        "Light Manufacturing"
95   HeavyMnfc        "Heavy Manufacturing"
96   TransComm        "Transport and Communication"
97   OthServices      "Other Services"
98   Energy           "Energy"
99 / ;

101 set mapk(commf,k) "Mapping from i to k" /
102   GrainsCrops      .GrainsCrops
103   MeatLstk         .MeatLstk
104   Extraction       .Energy
105   ProcFood         .ProcFood
106   TextWapp         .TextWapp
107   LightMnfc        .LightMnfc
108   HeavyMnfc        .HeavyMnfc
109   Util.Cons        .Energy
110   TransComm        .TransComm
111   OthServices      .OthServices
112 / ;

114 set lb "Land bundles" /
115   agr              "Agriculture"
116 / ;

118 set lb1(lb) "First land bundle" /
119   agr              "Livestock"
120 / ;

122 set maplb(lb,actf) "Mapping of activities to land bundles" /
123   agr              .Agriculture
124 / ;

126 * !!!! TO BE REVIEWED

128 set lb0 "Default land bundles" / lb1*lb1 / ;
129 set maplb0(lb, lb0) "Mapping of land bundles to original" /
130   agr.lb1
131 / ;

133 set wbnd "Aggregate water markets" /
134   N_A             "N.A"
135 / ;

137 set wbnd1(wbnd) "Top level water markets" /
138 / ;

140 set wbnd2(wbnd) "Second level water markets" /
141 / ;

```

```

143 set wbndEx(wbnd) "Exogenous water markets" /
144 / ;

146 set mapw1(wbnd,wbnd) "Mapping of first level water bundles" /
147 / ;

149 set mapw2(wbnd,actf) "Mapping of second level water bundle" /
150 / ;

152 set wbnda(wbnd) "Water bundles mapped one-to-one to activities" /
153 / ;

155 set wbndi(wbnd) "Water bundles mapped to aggregate output" /
156 / ;

158 set NRG "Energy bundles used in model" /
159   coa      "Coal"
160   oil      "Oil"
161   gas      "Gas"
162   ely      "Electricity"
163 / ;

165 set coa(NRG) "Coal bundle used in model" /
166   coa      "Coal"
167 / ;

169 set oil(NRG) "Oil bundle used in model" /
170   oil      "Oil"
171 / ;

173 set gas(NRG) "Gas bundle used in model" /
174   gas      "Gas"
175 / ;

177 set ely(NRG) "Electricity bundle used in model" /
178   ely      "Electricity"
179 / ;

181 set mape(NRG,e) "Mapping of energy commodities to energy bundles" /
182 * oil      .Extraction
183 * ely      .Util.Cons
184 / ;

186 * >>>> Sets required for 'growing' labor by skill

188 set skl(l) "Skill types for labor growth assumptions" /
189   SkLab
190 / ;

192 set elev / elev0+elev3 / ;

194 set educMap(r,l,elev) "Mapping of skills to education levels" ;

196 * Use GIDD definitions (i.e. "elev3" has no meaning)

198 educMap(r,"UnSkLab","elev0") $mapra("lmy",r) = yes ;
199 educMap(r,"SkLab","elev1") $mapra("lmy",r) = yes ;
200 educMap(r,"SkLab","elev2") $mapra("lmy",r) = yes ;

202 educMap(r,"UnSkLab","elev0") $mapra("hic",r) = yes ;
203 educMap(r,"UnSkLab","elev1") $mapra("hic",r) = yes ;
204 educMap(r,"SkLab","elev2") $mapra("hic",r) = yes ;

```

The subsets `elya` and `etd` are used by the power module. The first contains all of the (aggregate) power activities and the second contains the electricity transmission and distribution activity (normally a single activity). If these sets are empty, the distribution of power will be modeled using

the standard 'make' specification, i.e. a single CES nest for aggregation. An additional subset, `primElya`, is used post-simulation to calculate primary energy demand. The latter is composed of all combusted fossil fuels and primary electricity production, which excludes thermal power plants to avoid double counting.

The power module allows for a multiple nested CES structure for power aggregation. The example below, Listing D.6, shows the power nesting for a different map file. There are eight power activities (coal, oil, gas, nuclear, hydro, wind, solar, and other renewable), plus the transmission and distribution activity. The eight activities will be mapped to five power bundles (coal, gas, oil, nuclear and renewables), see set `pb`. The power aggregation is based on three nests, see figure 6—that is fully defined with the subsets in the power module of the map file. The subset `elyc` is designed to contain the single electricity commodity that is the output of aggregating electricity output across all electricity activities.

Listing D.6: A power bundle example

---

```

1  set elya(actf) "Power activities" /
2      clp
3      olp
4      gsp
5      nuc
6      hyd
7      wnd
8      sol
9      xel
10     etd
11 / ;

13 set etd(actf) "Electricity transmission and distribution activities" /
14     etd
15 / ;

17 set primElya(elya) "Primary energy power activities" /
18     nuc
19     hyd
20     wnd
21     sol
22     xel
23     etd
24 / ;

26 set pb "Power bundles" /
27     coap "Coal power"
28     gasp "Gas power"
29     oilp "Oil power"
30     nucp "Nuclear power"
31     othp "Other power"
32 / ;

34 set mappow(pb,elya) "Mapping of power activities to power bundles" /
35     coap.clp
36     gasp.gsp
37     oilp.olp
38     nucp.nuc
39     othp.(hyd, wnd, sol, xel)
40 / ;

42 set elyc(commf) "Electricity commodities" /
43     ely
44 / ;

```

---

The subsets `frt` and `feed` are used respectively by the `acr` and `lvs` activities to define the *ND2*

bundle.<sup>8</sup> The former contains the fertilizer commodities and the latter the feed commodities. The subset `iw` defines the commodities that can be bundled with the water factor in the water module. It could contain, for example, the water services commodity (`wtr`) from the GTAP database.

The subset `e` is a critical component of the energy module, see figures 5 and 8. The subset is also linked to the `NRG` set, the subsets `coa`, `oil`, `gas` and `ely` and the mapping set `mape`. Listing D.7 shows a potential configuration using the standard energy commodities in the GTAP database.<sup>9</sup> In the current configuration the set `NRG` and the name of the subsets `coa`, `oil`, `gas` and `ely` cannot be modified by the user. The subsets, however, can be empty. The subset `f` is not used by the model itself [TBC]. It is used at times in post-simulation to evaluate CO<sub>2</sub> emissions from the combustion of fuels. In the original GTAP database, this corresponds to the commodities `coa`, `oil`, `gas`, `p_c`, and `gdt`.

Listing D.7: An energy bundle example

---

```

1  set e(commf) "Energy commodities" /
2      coa
3      oil
4      gas
5      p_c
6
7      ely
8  / ;

10 set NRG "Energy bundles used in model" /
11     coa      "Coal"
12     oil      "Oil"
13     gas      "Gas"
14     ely      "Electricity"
15 / ;

17 set coa(NRG) "Coal bundle used in model" /
18     coa      "Coal"
19 / ;

21 set oil(NRG) "Oil bundle used in model" /
22     oil      "Oil"
23 / ;

25 set gas(NRG) "Gas bundle used in model" /
26     gas      "Gas"
27 / ;

29 set ely(NRG) "Electricity bundle used in model" /
30     ely      "Electricity"
31 / ;

33 set mape(NRG,e) "Mapping of energy commodities to energy bundles" /
34     COA.(coa)
35     OIL.(oil, p_c)
36     GAS.(gas)
37     ELY.(ely)
38 / ;

```

---

The mapping set `mapzf` is critical for the rural to urban migration module (if implemented). It would normally correspond to the `mapz` mapping set from the earlier part of the mapping file—but corresponding to the model aggregation, not the data aggregation. If the migration elasticity is set to infinity, the model will assume perfect labor mobility and the `mapzf` mapping set will be ignored.

<sup>8</sup> From the point of view of the code, these are used to determine the mapping sets `mapi1` and `mapi2`.

<sup>9</sup> The ordering in the map file is somewhat arbitrary and mostly up to user discretion, unless there are specific dependencies.

The top level demand nest for consumer demand is based on a different set of goods than the commodities defined by the set  $i$ . This allows for a 'make' or 'transition' matrix approach to consumer demand specification, see figure 7. The household commodities are specified over the set  $k$ . It is simple mapping from the supply commodities ( $i$ ) to consumed commodities ( $k$ ). If there is a one-to-one mapping, the consumer make matrix is fully diagonal. At the moment, one of the key purposes of the make matrix is to allow for a consumer energy bundle, with a structure similar to the energy bundle in production, see figure 8. It could however be used to construct other non-diagonal elements. For example crop commodities could be combined into a single consumer crop bundle and the livestock commodities could be combined into a single consumer livestock bundle. The subsequent CES nests would then allocate the top level demand to the relevant components. Listing ?? provides an example of a consumer mapping for energy only.<sup>10</sup>

Listing D.8: An example of the consumer 'make' aggregation

---

```

1  set k "Household commodities" /
2      crp  "Cereals"
3      osd  "Oil seeds"
4      xcr  "Other crops"
5      lvs  "Meat and wool"
6      rmk  "Raw milk"
7      frs  "Forestry"
8      fsh  "Fisheries"
9      omn  "Other mining"
10     met  "Meat products"
11     vol  "Vegetable oils"
12     mil  "Dairy products"
13     ofd  "Other foods products"
14     bt   "Beverages and tobacco products"
15     tex  "Textiles"
16     wap  "Wearing apparel"
17     lea  "Leather products"
18     wdp  "Lumber, paper and paper products"
19     crp  "Chemicals, rubber and plastics"
20     mmn  "Basic metals and minerals"
21     ele  "Electronic equipment"
22     mvh  "Motor vehicles"
23     xma  "Other industry & manufacturing"
24     cns  "Construction"
25     ttp  "Trade and transport"
26     bsv  "Financial and business services"
27     dwe  "Dwellings"
28     xsv  "Other services"
29     nrg  "Energy bundle"
30 / ;

32  set mapk(commf,k) "Mapping from i to k" /
33      cer.cer
34      osd.osd
35      xcr.xcr
36      lvs.lvs
37      rmk.rmk
38      frs.frs
39      fsh.fsh
40      coa.nrg
41      oil.nrg
42      gas.nrg
43      omn.omn
44      met.met
45      vol.vol
46      mil.mil
47      ofd.ofd

```

---

<sup>10</sup> Like for the other activities and commodities, the aggregation facility will automatically append the '-k' suffix to consumer labels.



```

48     b_t.b_t
49     tex.tex
50     wap.wap
51     lea.lea
52     wdp.wdp
53     p_c.nrg
54     crp.crp
55     mmn.mmn
56     ele.ele
57     mvh.mvh
58     xma.xma
59     ely.nrg
60     cns.cns
61     ttp.ttp
62     bsv.bsv
63     dwe.dwe
64     xsv.xsv
65 / ;

```

The land module depends on user-determined land bundles, see figure 10. The model allows for a fair amount of user flexibility. There are three nests, but one of the nests allows for a variable number of land bundles. The mapping of the agricultural activities to the land bundles is also determined by the user. Listing D.9 shows an example of a land bundle that corresponds to that used in the MAGNET model.<sup>11</sup>

The MAGNET specification has only three bundles (L, FCP and COP). We have arranged these into five bundles. The top is split between the `hrt` bundle and all of the rest (that corresponds to the FCP bundle). We assume that the top level CET elasticity in MAGNET,  $\sigma^1$  holds for the top level bundle in ENVISAGE but also for the bundle composed of activities in the `hrt` bundle. This has the same impact as mapping directly the `hrt` activities to the top level bundle as in MAGNET. The `XNLB` bundle will then be composed of the `lvs` and `cer` bundles. MAGNET's CET elasticity,  $\sigma^2$  is then applied to the `XNLB` and `lvs` bundles, which has the same impact as mapping directly the `lvs` activities to the `XNLB` bundle as in MAGNET. Finally, MAGNET's  $\sigma^3$  elasticity is applied to the `cer` bundle. The set `lb0` is used to map the elasticities in the parameter file to the model elasticities—the set is not part of the the model. [This mapping is still under review and users can override these in the model parameter file].

Listing D.9: An example of land configuration

```

1  set lb "Land bundles" /
2     hrt      "Horticulture and other crops"
3     lvs      "Livestock and sugar"
4     cer      "Cereals and oil seeds"
5  / ;

7  set lb1(lb) "First land bundle" /
8     hrt      "Horticulture and other crops"
9  / ;

11 set maplb(lb,actf) "Mapping of activities to land bundles" /
12     hrt.v-f
13     htr.ocr
14     lvs.lvs
15     lvs.sug
16     cer.wht
17     cer.gro
18     cer.osd
19 / ;

21 set lb0 "Default land bundles" / lb1*lb3 / ;

```

<sup>11</sup> <http://www.magnet-model.org/MagnetModuleDescription.pdf>, page 74.

---

```

22 set maplb0(lb, lb0) "Mapping of land bundles to original" /
23   hrt.lb1
24   lvs.lb2
25   cer.lb3
26 / ;

```

---

The water module has three basic bundle types:

1. Water bundles that have activities mapped to them—for example irrigation water in crop activities
2. Water bundles that are linked to aggregate output indices—for example industrial water use
3. Exogenous bundles not linked directly to the economy—for example ground water recharge or water for environmental services.

These fundamental water bundles are structured in a nested CET nest for which users have significant control. In the example in Listing D.10 there is one bundle of the first type (crops), three bundles of the second type (livestock, municipal and industrial) and two bundles of the third type (environmental services and ground water recharge). There are two intermediate bundles that form the top level CET nest—agriculture and non-agriculture, see figure 11. The set `wbnd` defines all of the possible bundles. The set `wbnd1` defines the top level bundles—in principle there could be more than two. The set `wbnd2` defines the second level bundles. The set `wbndEx` defines the exogenous bundles. These will be subtracted from total water supply. The mapping set `mapw1` maps fundamental bundles to the top-level intermediate water bundles. The mapping set `mapw2` maps all activities to one of the fundamental bundles. The subset `wbnda` indicates which water bundles are mapped directly to activity-based water demand. With the current water database, only irrigated crops use water directly. The subset `wbndi` indicates which water bundles are mapped to output indices. The elasticities for the water module need to be provided in the user-built parameter file—see below.

Listing D.10: An example of water configuration

---

```

1 set wbnd "Aggregate water markets" /
2   agr "Agriculture"
3   nag "Non-agriculture"
4   env "Environmental services"
5   grd "Ground water recharge"
6   crp "Crops"
7   lvs "Livestock"
8   ind "Industrial use"
9   mun "Municipal use"
10 / ;

12 set wbnd1(wbnd) "Top level water markets" /
13   agr "Agriculture"
14   nag "Non-agriculture"
15 / ;

17 set wbnd2(wbnd) "Second level water markets" /
18   crp "Crops"
19   lvs "Livestock"
20   ind "Industrial use"
21   mun "Municipal use"
22 / ;

24 set wbndEx(wbnd) "Exogenous water markets" /
25   env "Environmental services"
26   grd "Ground water recharge"

```

---

```

27 / ;

29 set mapw1(wbnd,wbnd) "Mapping of first level water bundles" /
30   agr.(crp,lvs)
31   nag.(mun,ind)
32 / ;

34 set mapw2(wbnd,actf) "Mapping of second level water bundle" ;
35 mapw2("crp",acr) = yes ;
36 mapw2("lvs",alv) = yes ;
37 mapw2("ind",man) = yes ;
38 mapw2("mun",actf)$ (not (acr(actf) or alv(actf) or man(actf))) = yes ;

40 set wbnda(wbnd) "Water bundles mapped one-to-one to activities" /
41   crp      "Crops"
42 / ;

44 set wbndi(wbnd) "Water bundles mapped to aggregate output" /
45   lvs      "Livestock"
46   ind      "Industrial use"
47   mun      "Municipal use"
48 / ;

```

---

The remaining section of the map file relates to the use of the IIASA- or GIDD-based education projections to shape the growth of skilled and unskilled labor. This is not fundamental to the model, but is part of defining the dynamic scenario.

One subset defines skilled labor, `sk1`. This is different from the `u1` and `s1` subsets that are used to allocate labor between the *LAB1* and *LAB2* bundles. The purpose here is only to drive the relative growth of unskilled and skilled labor. The mapping set, `educMap`, determines the mapping between labor skills and the education projections. There are two different education projections. The IIASA education projections have four classifications: none, primary, secondary and tertiary (labeled `ENONE`, `EPRIM`, `ESECN` and `ETERT` in the SSP database, but labeled `ELEVO*ELEV3` in the scenario file.) The GIDD database has three classifications: 0-6 years of education, 6-9 years of education and 9 or more years (labeled `EDUC0_6`, `EDUC6_9`, and `EDUC9UP` in the GIDD database, but labeled `ELEVO*ELEV2` in the scenario file.) If using the IIASA database, all four education levels should be mapped to the skills. If using the GIDD database, only three of the levels are active. The mappings can vary across regions. For example, in high-income countries, skilled workers might be only tertiary and above. In developing countries, skilled workers may also include workers with a secondary education.

## Running the aggregation

The aggregation is coded in GAMS and can be run in the GAMS IDE or from a Windows command console. For the latter, the command line is:

```

gams aggGTAP --basename=[basename] --ifAlt=[OFF|ON] --model=[GTAP|ENV]
      --ifCSV=[0|1] --ifAggTrade=[0|1]

```

The aggregation routine is named `AggGTAP.gms`. The user needs to provide the base name of the project, for example `10x10`. The aggregation facility will prepare additional output if the user wishes to use `AlterTax`. In that case, the `ifAlt` setting should be set to `ON`, otherwise it should be set to `OFF`. Aggregation is handled somewhat differently for the GTAP and ENVISAGE models. The `model` option should take the value `GTAP` for the GTAP model, and `ENV` for the ENVISAGE model. The `ifCSV` option takes either the value 0 or 1. If it is set to 1, the aggregation facility will output the SAM, energy and emissions data in a CSV formatted file that can be uploaded into Excel. The

`ifAggTrade` option takes either the value 0 or 1. If it is set to 1, the output SAM will collapse bilateral trade to a single trade account, otherwise the SAM will have the full bilateral trade (on both the import and export side).

If the aggregation is intended for the ENVISAGE Model, the `aggGTAP` routine will invoke the `convertLabel` program that will convert the labels in the parameter file to the labels needed for the model.

#### D.1.4 Filtering

The filtering algorithm was developed by Tom Rutherford to remove small value flows from the aggregated database (Lanz and Rutherford (2016)). The distributed version has been modified by Wolfgang Britz to handle some additional features. The user needs to prepare a very short include file with the algorithms's options (`[basename]Flt.gms`). Listing D.11 is an example of the filter options file.

##### The filter options file

The consecutively applied filtering and rebalancing approach is an extension of the method and code developed by Tom Rutherford.<sup>12</sup> It deletes components of the SAM depending on their shares on specific totals, according to the 'Relative tolerance' (`relTol`):

- Domestic and imported intermediate demand of a commodity are dropped relative to its total output
- Private/government/investment domestic respectively import demand of a commodity are dropped relative to total Private/government/investment domestic respectively import demand
- Trade flows of a product are dropped if both shares on total exports of that product and its exporter and on imports of that product and its importer are below the relative threshold
- Production is dropped if net production of a commodity, i.e. after intermediate use of that commodity in its own production is deducted, is below the relative threshold with regard to total net production

Listing D.11: An example of a filter file

```

1  scalars
2  ifKeepIntermediateConsumption / 1 /
3  ifKeepPrivateconsumption      / 1 /
4  ifKeepGovernmentconsumption   / 1 /
5  ifKeepInvestments              / 1 /
6  ifGDPKeep                      / 1 /
7  ifKeepFactorincomeplusbop     / 1 /
8  ifAdjDepr                       / 1 /
9  abstol                          / 1e-10 /
10 relTol                          / 0.005 /
11 relTolRed                       / 1e-6 /
12 nsteps                          / 5 /
13 minNumTransactions             / 50000 /
14 ;

```

<sup>12</sup> The following explanations are adapted from Britz's documentation. See [http://www.ilr.uni-bonn.de/em/rsrch/cgebox/cgebox\\_GUI.pdf](http://www.ilr.uni-bonn.de/em/rsrch/cgebox/cgebox_GUI.pdf).

---

```

16 file log / %baseName%flt.log / ;
17 put log ;

19 $ontext
20 $setglobal excRegs
21 $setglobal excSecs
22 $offtext

24 $setglobal excSecs "sol, wnd, xel"
25 $setGlobal excCombined 1

```

---

The absolute tolerance level, `absTol`, deletes any trade flow value that is below the tolerance level in absolute terms. With a value of 1.E-10, that preliminary deletion step is skipped. It is generally not recommended to use absolute deletion thresholds above 1.E-6 in combination with rebalancing as the subsequent relative thresholds will anyhow apply more refined rules.

The filtering process imposes restrictions which should maintain the regional SAMs balanced. Additional constraints ensure that production activities require added value and intermediate inputs, if not already otherwise found in the data base. As filtering systematically removes elements from the SAM and the trade matrices, the process implies without further corrections shrinking the economies. During rebalancing, the algorithm can therefore add penalties for deviations from the following aggregate transactions:

- Intermediate consumption
- Private consumption
- Public consumption
- Investment
- Factor income plus BOP
- GDP

The imposition of these penalties is driven by the relevant flags in the filter options file. By adding these penalties terms, the non-deleted entries (and thus most important transactions) tend to be scaled upwards. It is generally recommended to use these penalties terms. The code will also scale all non-deleted trade flows to approximately maintain the total volume of international trade and related international transport margins.

The absolute and relative thresholds are stepwise enforced. For the first few steps, exponential increases are used, starting with minus half the number of steps. For six steps, to give an example, the first thresholds applied will be the 1.E-3 of the final one, next 1.E-2 and finally 10%. The remaining steps will use equal linear increases between 10% and the desired final ones. Once the final thresholds are active, filtering is still applied several times until no small values are found any longer. The code should ensure that the resulting transactions are still fully consistent with each other, i.e. both the resulting trade matrices and the SAMs are balanced. The changes imposed by filtering and subsequent balancing are stored in the log file. Inspecting how the stepwise enforcement of the thresholds impacts on the number of non-zero items can inform on an appropriate level for tolerances to be used.

The SAMs used during filtering are—as in the GTAP database—defined in Million dollars. An absolute threshold of 1.E-6 will hence delete any economic transactions worth a single dollar or less. In SAMs with high regional and sectoral detail, even such tiny transactions might make up

to 10% of the non-zero entries. Increasing the threshold for \$1000 might remove 1/4 or more of all non-empty transactions. Similar results are found from using relative tolerances of 0.001%.

Thanks to balancing, also rather dis-aggregated versions of the model with large number of sectors and regions can be used. The biggest impact of the filtering is typically on transactions related to bilateral trade flows. Here, often 50% or more of the flows account for only 1% of the total value of these transactions. Thus, tiny changes in the relative tolerance can have a considerable impact on the number of deleted transaction, and one might need to experiment with settings in the range around 1.E-1 to 1.E-4 to find a compromise between sparsity and the implied changes on structure of the economy. For very large data sets (e.g. a 1:1 version) filtering thresholds above 1% might be needed to yield reasonable model sizes. The user can additionally define a minimum number of transactions to be kept, which reduces the need to experiment with different thresholds as the filtering process will stop once less than the desired number of transactions is reached. Tests with the model have shown that the model in full resolution of the GTAP 8.1 data base without filtering, i.e. 57 sectors and 134 regions, can be solved in partial trade liberalization scenarios, solution has failed with other shocks on models with more than 400,000 transactions, especially if the global bank mechanism active. A close look at the filtering statistics is recommended, to avoid sharp impacts on the structure of the economy. A more detailed discussion on the relation between model dis-aggregation, filtering, solution behavior and simulated welfare impacts is provided in [Britz and van der Mensbrugge \(2016\)](#).

**Special treatment for specific regions and sectors** When building a data base for a project, it might be desirable to apply less aggressive filtering thresholds for specific regions and/or sectors in the focus of the application. The algorithm therefore allows defining lists of regions/sectors with accompanying specific thresholds. The codes for regions/sectors needs to be inputted in the global defines called `excRegs` and `excSecs`. 'Reduced thresholds only in combination' will apply the different threshold only to the intersection of the inputted regions and sectors (`excCombined`), otherwise, all regions and sectors inputted will be receive different thresholds. Take an example where you enter for regions `xoc` and for sectors `pdr`. If 'Reduced thresholds only in combination' is NOT switched on, all transactions of the region `xoc` and all transactions for the sector `pdr` will be treated differently. If the 'Reduced thresholds only in combination' is active, only the transaction relating both to `pdr` and the region `xoc` are exemptions. However, filtering for the remaining sectors/regions has still an impact on these exemptions. For example, if production of a sector in a region is dropped, the related export flows need to be dropped as well, affecting potentially transactions in regions and sectors where tighter thresholds are used. Tests have however indicated that very few transactions are lost in regions/sector where stringent thresholds are applied as long as the overall filtering thresholds are not too aggressive.

**Diagnostics** The filter listing file (`filter.lst`) and the log file (`[basename]flt.log`) provide a number of useful diagnostics regarding the filtering process. Users can also load the resulting 'CSV' file that contains the SAM and energy/emission values both pre- and post-filtering.

### Running the filter program

The filter routine is coded in GAMS and can be run in the GAMS IDE or from a Windows command console. For the latter, the command line is:

```
gams filter --basename=[basename] --ifCSV=[0|1] --ifAggTrade=[0|1]
```

The filter routine is named `filter.gms`. The user needs to provide the base name of the project, for example `10x10`. The `ifCSV` option takes either the value 0 or 1. If it is set to 1, the aggregation facility will output the SAM, energy and emissions data in a CSV formatted file that can be uploaded into Excel. The `ifAggTrade` option takes either the value 0 or 1. If it is set to 1, the output SAM will collapse bilateral trade to a single trade account, otherwise the SAM will have the full bilateral trade (on both the import and export side).

### D.1.5 Altering a database

The third module in the data preparation routine is called `Altertax`, first developed by [Malcolm \(1998\)](#) for the GTAP model. The `Altertax` module is optional. It allows the user to make changes to the structure of the database that minimizes the distortions from the original database. It is typically used to change tax rates, for example import tariffs. The distributed version of `Altertax` relies on a version of the GTAP model written in GAMS.<sup>13</sup> The main thrust of `Altertax` is to convert most of the model elasticities to 1 thereby all CES functions are essentially converted to Cobb-Douglas functional forms. The latter have the property of conserving value shares. In addition, the `Altertax` version of the GTAP model assumes a Cobb-Douglas utility function for private consumption. A fixed current account is also imposed for the balance of payment closure. The overrides are provided in the `AlterTax.gms` file.

#### Option file

If running `Altertax`, the user needs to prepare an option file that specifies the nature of the alteration—it is to be named `[basename]Alt.gms`. The file can be empty, in which case `Altertax` would simply re-balance the initial database—though this will already have been done in the filter routine if it was used. [Listing D.12](#) shows a very simple change: a rise of 40% on the import tariff of motor vehicles in Argentina. One could of course overlay the entire import tariff schedule for Argentina in this file—including any bilateral dimensions. Note that the activity and commodity labels will need the appropriate suffix.

Listing D.12: An example of a filter file

---

```
2  imptx.fx(s, "mvh-c", "arg", tsim) = 1.4*imptx.l(s, "mvh-c", "arg", tsim) ;
```

---

A more complicated, though artificial example is provided in [Listing D.13](#). This example shows how to phase in the shock to ease numerical convergence. The shock is a 50% cut in all tariffs. The user provides the number of iterations to phase in the shocks, say for example 4. The iteration count is first checked to see if it is 1, in which case there is no iteration phase and the shock is imposed in one shot. If the iteration count is greater than 1, the shock will be phased in. The `solver` needs to be invoked for the first  $n - 1$  iterations, the  $n^{\text{th}}$  iteration will be solved in the standard (and final) invocation of the solver.

Listing D.13: An example of a filter file

---

```
1  *
2  *  Altertax shock
3  *
4  *  If new policy needs to be phased in, the basic implementation is
```

---

<sup>13</sup> The GAMS-based GTAP model has been developed by GTAP staff and documentation is available upon request. It differs from the Rutherford *GTAPinGAMS* model ([Lanz and Rutherford \(2016\)](#)) as it is intended to exactly replicate the specification of the standard GTAP model in *GEMPACK* ([Hertel \(1997\)](#)).

---

```

5 *
6 *   p(it) = pfinal*(it/n) + pinitial*(1-it/n)
7 *
8 *   See below for an example
9 *
10 *   Set the number of iterations on the command line, for example --niter=4

12 *   Cut initial tariffs by 50%
13 *   -----

15 if(niter(tsim) eq 1,

17 *   No phase in of cuts

19   imptx.fx(r,i,rp,tsim) = 0.5*imptx.l(r,i,rp,"base") ;

21 else

23 *   Phase in the shock

25   for(iter=1 to niter(tsim),
26     imptx.fx(r,i,rp,tsim) = 0.5*imptx.l(r,i,rp,"base") * (iter/niter(tsim))
27     + imptx.l(r,i,rp,"Base") * (1 - iter/niter(tsim)) ;
28     if(iter < niter(tsim),
29       $batinclude "solve.gms" gtap
30     ) ;
31   ) ;
32 ) ;

```

---

## Running Altertax

The Altertax routine is coded in GAMS and can be run in the GAMS IDE or from a Windows command console. For the latter, the command line is:

```
gams AlterTax --BaseName=[basename] --niter=1 --ifCSV=[0|1] -idir=GTAPModel
```

The filter routine is named `Altertax.gms`. The user needs to provide the base name of the project, for example `10x10`. The user needs to provide the number of iterations for a phase-in of the shock. For most modest shocks, a value of 1 should be sufficient. Note that it is up to the user to make sure the Altertax option file contains the necessary code for the phase in of the shock. The `ifCSV` option takes either the value 0 or 1. If it is set to 1, the aggregation facility will output the SAM, energy and emissions data in a CSV formatted file that can be uploaded into Excel. The `iDir` option provides a pointer to the folder containing the GTAP model code. Under default configurations the model code is a sub-folder in the *Data* directory with the name *GTAPModel*.

If the `ifCSV` option is set to 1, the SAM and energy/emissions data will be output in a CSV-formatted file for input into Excel. The CSV file will have a 'time' dimension that takes three values. 'Year' equal to 1 corresponds to the base year initialization of model variables and parameters and should correspond to the input database. 'Year' 2 corresponds to a first simulation with no shock. It should re-produce the base data as well, i.e. the results in 'Year' 1 and 2 should be identical to within a very small tolerance level. 'Year' equal to 3 will correspond to the post-shock structure of the database and comparison with either 'Year' 1 or 2 will highlight deviations from the initial database.



### D.1.6 Integrated command file

The distribution is delivered with a Windows command file, `makeData.cmd`, that can be used to run all of the modules in sequence. To run the command, type the following in a Windows console:

```
makeData [baseName] [-ifFilter] [-ifAlt] [-ifEnv]
```

This will invoke the `makeData` command file. There are three options. To invoke the filter module, enter the `-ifFilter` option on the command line. To invoke the Altermat module, enter the `-ifAlt` option on the command line. To prepare an aggregation for the ENVISAGE Model, enter the `-ifEnv` option on the command line. N.B. We have made every attempt to make the `makeData` command file as robust as possible—but coding a Windows command file is not for the faint-hearted. We would be happy to have any feedback on its robustness and usefulness.

The `makeData` command file automates the copying and moving of files. In a first step, it will create a folder with the name of the aggregation code, `[basename]` if it does not exist. It will also create the four sub-folders: `Agg`, `Flt`, `Alt` and `Fn1`. The aggregation routine creates 10 GDX files (and optionally the CSV file). The filter routine only modifies the five data files: `[basename]Dat.gdx`, `[basename]Vole.gdx`, `[basename]Emiss.gdx`, `[basename]NC02.gdx` and `[basename]Wages.gdx`. The `makeData` command file copies the other (non-modified) files from the `Agg` folder to the `Flt` folder. Similarly for Altermat, the command file will copy the non-modified files from the `Flt` folder to the `Alt` folder. If the respective routines are not invoked, the command file will copy ALL files from one folder to the next. In a final step, all the GDX files in the `Alt` folder will be copied to the `Fn1` folder from where the user can copy the final data and parameter files to the working folder for subsequent simulations.

## D.2 Model Simulations

### D.2.1 Introduction

We introduce in this section how to run ENVISAGE model simulations. There are three fundamental types of simulations:

- Comparative static. Comparative static involves introducing a shock to the equilibrium database with no dynamic elements—such as factor accumulation, technology and preference changes, etc. (of course a comparative static shock could include a shock to one of these elements). The key difference between the comparative static and dynamic version of the model is the specification of the capital account as described in the model description. Capital markets close with a CET transformation function in the comparative static version, which also has no vintage capital. In the dynamic version, installed capital is assumed to be only partially mobile across activities and the model explicitly incorporates vintages.
- Recursive dynamic with baseline calibration. This version of the model is dynamic and the model is used to determine some model parameters subject to some targets. For example, GDP growth may be exogenous and labor productivity is calibrated to achieve a given growth target, and/or the investment to GDP ratio is targeted and the household savings rate is adjusted to meet the target, etc.

- Recursive dynamic shock scenario with pre-calibrated dynamic parameters. This type of dynamic scenario will use the results from the baseline calibration scenario for some of the underlying dynamic trends, for example labor productivity. In the absence of a shock, this type of simulation should re-produce the baseline scenario.

There is no single best way to run model simulations—in either comparative static or dynamic mode as GAMS provides great flexibility. We introduce herein a unified system that can work for all three types of simulations. This system is useful because it highlights the differences across the simulation types. It does have some potential drawbacks that will be highlighted below.

## D.2.2 Preliminaries

The model code is composed of a series of GAMS file, this helps with modularizing the code. Typically, all of the core model code will be available in a single folder and the user will set the `idir` option in GAMS to the location of the core model code, say for example in the *Model* directory.

Table D.6 is a list of the files that constitute the core model code. The model’s declarations and equation specification are contained in the `model.gms` file.

Table D.6: Distributed model files

<i>File name</i>	<i>Description</i>
<code>cal.gms</code>	Calibration of model parameters
<code>closure.gms</code>	Default closure assumptions
<code>compScen.gms</code>	'Scenario' file for comparative static simulatons
<code>init.gms</code>	Initialization of model variables
<code>initScen.gms</code>	Scenario file for dynamic simulations
<code>InitVar.gms</code>	Inter-period initialization of model variables
<code>initVint.gms</code>	Second period initialization of vintage volume variables
<code>iterloop.gms</code>	Inter-period code
<code>miscDat.gms</code>	Currently this file contains only an energy conversion table. It is used post-simulation.
<code>model.gms</code>	Core model specification
<code>postsim.gms</code>	Post-simulation statements—mostly creation of output CSV files
<code>recal.gms</code>	Inter-period core code to update vintage technology parameters and potentially other dynamically calibrated parameters such as the Armington preference parameters
<code>recalnnn.gms</code>	Code that updates technology parameters such as <b>and1</b>
<code>recalnnt.gms</code>	Code that updates technology parameters such as <b>aland</b>
<code>recalnrg.gms</code>	Code that updates technology parameters such as <b>aNRG</b>
<code>recalvat.gms</code>	Code that updates technology parameters such as <b>axp</b>
<code>recalvnn.gms</code>	Code that updates technology parameters such as <b>ava</b>
<code>recalvnt.gms</code>	Code that updates technology parameters such as <b>ak</b>
<code>recalxanrg.gms</code>	Code that updates technology parameters such as <b>aeio</b> with no energy nesting
<code>recalxanrgn.gms</code>	Code that updates technology parameters such as <b>aeio</b> with energy nesting
<code>sam.gms</code>	Code that writes out the simulation SAMs in CSV format
<code>SaveParm.gms</code>	Code that writes out key parameters
<code>scale.gms</code>	Code that scales model variables/equations. Currently not used.
<code>solve.gms</code>	Code that invokes the solver.

Figure 14 provides a schematic view of a simulation. In the current setup, the user prepares a project specific file with the name `[basename]Opt.gms`. This file contains the common options and

statements across all simulations in the project. It is designed to handle both comparative static and dynamic simulations. Many of the global options can be over-ridden in subsequent simulation files (for example `runSim.gms`), however, it is normally not possible to override set definitions such as time. The first step in a simulation file, such as `runSim.gms` is to read the options file. The options file will also load a number of the core GAMS code and some user files. The 'Sets' file, which is user-based, is generated by the aggregation facility and normally requires no further input. The parameter file, described below [tbd] does need user input for each project. The core code that is read includes the model specification (`model.gms`), model initialization (`init.gms`), model calibration (`cal.gms`) and the default closure rules (`closure.gms`). After defining the default closure the model is ready to be solved.

The model is solved over time—even in the case of comparative static simulations—albeit skipping the first period, which will contain the initialized and calibrated solution using the base data. A number of statements are executed at the beginning of each period that initializes variables, may recalibrate technology parameter (to be described), updates time-based shocks in the case of dynamic scenarios and introduces shocks for both comparative static and dynamic simulations. Once all of the period updating is finished, the solver is invoked. The simulation file will loop over all time periods—as long as the model converges. After looping over all periods, model results will be saved—optionally in a CSV file and always in GDX format.

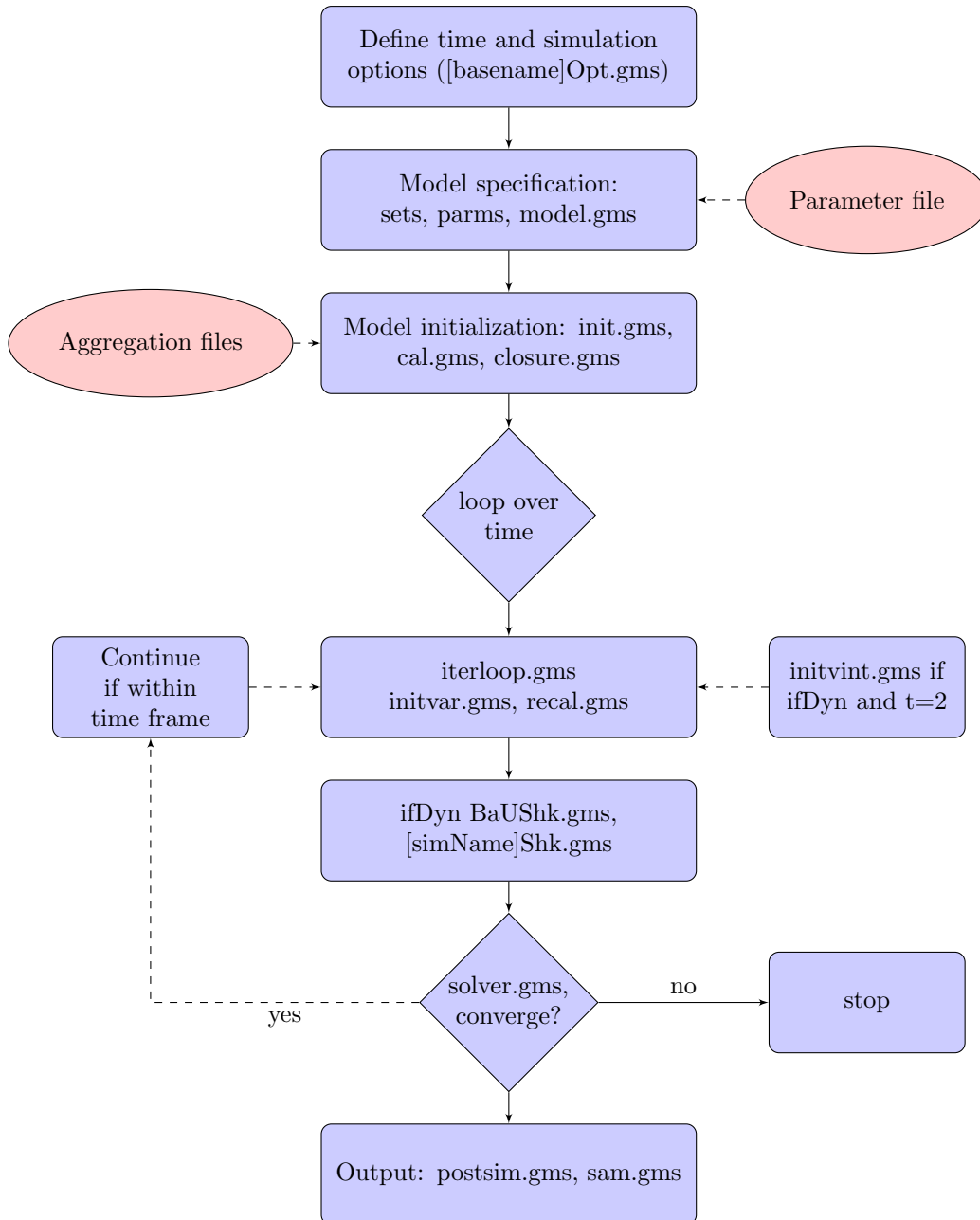
In a typical application, the user will create a project directory with the base name of the project. The simulation files will be contained in that directory. The distribution comes with several files that provide examples of how to run both comparative static and dynamic simulations. Beyond the aggregated data files, the user needs to prepare a file with the model parameters. This file can be fairly generic if based on an aggregation of one of the existing parameter files. However, there a number of options that users need to define to complete the model specification—such as the labor market closure.

### The user-based parameter file

While most of the inputs to the model are prepared by the aggregation facility, it is up to the user to prepare the file with project-specific model parameters. The file has the name `[basename]Prm.gms`. There is a generic parameter file that simply initializes all model parameters to those generated by the aggregation facility. These can be overridden—after the ones from the aggregation facility are loaded, but before the initialization of the model parameters. The read-in parameters will have a suffix of '0'. The model parameters have no suffix.

The aggregated parameters reflect values used by the OECD's Environment Directorate Env-Linkages Model (Chateau, Dellink, and Lanzi (2014)). The Envisage Model has some new features not fully reflected in the original Env-Linkages Model, such as the power and water module. The file does include some initial levels for the power elasticities, but [for the moment] does not include the elasticities for the water module. These must be entered by the user if the water module is active. Another critical set of elasticities is the natural resource supply elasticities. [NEW] It is the user's responsibility to enter these elasticities (for the original model aggregation). The user must enter a pair of elasticities for each natural resource and for all regions. The parameter to be initialized is `etanrfx0` that takes three indices: region, activity and a special one that takes the values of 'lo' and 'hi'. This latter is represented by the set `lh`. These refer to the supply elasticity to apply as a function of market conditions. The 'lo' supply elasticity is to be used for a market under contraction. The 'up' supply elasticity is to be used for an expanding market. Intuition suggests that the 'lo' elasticity will be greater than the 'hi' elasticity, i.e. it is easier to contract supply than to expand it. These should be carefully scrutinized and potentially adjusted during

Figure 14: Simulation flow chart



baseline simulation runs. [NEW] The user must enter the investment allocation elasticities to be used with the flexible capital flow specification. The initial elasticities are to be initialized with the parameter `epsRor0`. The default value for the GTAP model is 10.

The comparative static version of the model uses a weighted average of the vintage-dependent elasticities. Users are free to change the weights, where the default values are 0.8 for *Old* and 0.2 for *New*.

There are a number of other key assumptions that are included in the parameter file. The first relates to the labor market assumptions. The user must initialize the matrix of options in the parameter `labHyp`. It has a regional and skill index and 11 attributes described below.

1. `omegam`. This determines labor market segmentation. A finite value will implement labor market segmentation—typically defined over agricultural versus non-agricultural activities. Perfect labor mobility is implemented if the value is infinity (`INF`). If it is finite, this will be the value of the labor mobility elasticity in equation (G-1),  $\omega^m$ , the elasticity of rural to urban migration with respect to the expected urban wage premium.
2. `migr0`. This determines the initial level of migration as a percent of the rural labor force. For example, if it is set to 1.0, the initial level of rural to urban migration will be set to 1% of the rural labor force. It is ignored if the migration elasticity is infinity.
3. `uezRur0`, `uezUrb0`. These parameters provide the initial unemployment rate (in percent) in respectively the rural and urban labor markets. In the case of full labor mobility, only the urban unemployment rate is active and the rural rate is ignored.
4. `uezminzRur0`, `uezminzUrb0`. These parameters provide the lower bound for the unemployment rate (in percent) in respectively the rural and urban labor markets. The rural rate is ignored in the case of full labor mobility. An error will be issued if the initial unemployment rate is less than the minimum unemployment rate.
5. `resWageRur0`, `resWageUrb0`. These parameters determine the initial level of the reservation wage with respect to the prevailing wage. A value of 1 indicates that the reservation wage is binding and the the unemployment rate is greater than (or equal to) the minimum unemployment rate. A value less than one indicates the distance between the initial equilibrium wage and the reservation wage. The rural reservation wage is ignored in the case of full labor mobility. A value of `NA` indicates full employment (at all times). An error will be issued if the value is greater than 1.
6. `omegarwg` `omegarwue` `omegarwp`. The reservation wage is a function of three indicators—the growth of per capita GDP, the unemployment rate and the CPI. The elasticity is positive for income growth and the CPI. The reservation wage is negatively related to the unemployment rate, i.e. an increase in unemployment would tend to dampen the reservation wage. To avoid problems with division by zero, the relationship between the reservation wage and unemployment is converted to a relationship between the reservation wage and the employment rate, i.e.  $(1 - UE)$ . Thus if the elasticity of the reservation wage is  $\omega^{ue}$ , the elasticity of the reservation wage with respect to the employment rate is  $-\omega^{ue} (1 - UE) / UE$ . The user is expected to enter the elasticity with respect to the employment rate and it should be a positive number.

There are three additional parameters that allow for control of the dynamics of the capital market. The first, `cap_out_Ratio0`, allows for overriding the initial capital to GDP ratio. The

initial capital stock is an estimate derived from the Penn World Tables.<sup>14</sup> For some countries and/or regional aggregations the estimates can lead to significant jumping off problems in the initial years. A value of NA will ignore the override. The second is the depreciation rate, `deprT`, which for some rapidly growing countries is too low. The default GTAP value is 4%. The third is an investment target, `invTarget0`. Users can input this for particular years, e.g. 2030 and 2050. Code in the 'Opt' file will translate these targets into linear changes for the investment target, for example between 2011 and 2030 and then between 2030 and 2050, in order to smooth the path between target years.

The final section allows the user to implement 'twists' to the Armington preference parameters. There are three sets of twist parameters and one twist-related regional mapping set. The top-level Armington twists require two different parameters depending on the value of `ArmFlag`. In the case of national sourcing, i.e. when `ArmFlag` = 0, the relevant twist parameter is `tw1`, which is region-, commodity and time-specific. With agent-based sourcing, i.e. when `ArmFlag` = 1, the relevant twist parameter is `tw1`, that is in addition agent-specific. A positive value, for example 0.02, would lead to a change in the ratio of imports to domestic demand of 2 percent—assuming constant prices. The parameter `tw2` is applied to the second level nesting. The  $r$  index refers to the importing country. The twist can be applied to one single region, or a group of regions. The user designates the targeted exporters in the regional mapping set `rtwtgt`. The first regional index in the mapping set refers to the targeted exporters and the second regional index refers to the importer. For example, to increase the import shares of China, Japan and Korea in the U.S. using the twist, the relevant mapping is `(chn,jpn,kor).usa`. Note that in this implementation, the same twist is applied uniformly across the targeted exporters.

### D.2.3 Common options

The first set of global options are entered as definitions through GAMS' `$setGlobal` statement. The following enumerates the options:

1. **wDir**. By default this is set to the active directory.
2. **SSPMOD**. This option defines the choice of the economic model used for the SSP GDP projections. There are currently two choices: `OECD` and `IIASA`.<sup>15</sup>
3. **SSPSCEN**. This option defines which SSP to use for the GDP projections (and will be combined with the model choice). There are five valid options: `SSP1-SSP5`.
4. **POPSCEN**. This option defines which population projection to use. There are nine valid options. The IIASA SSP population projections are labeled `SSP1-SSP5`. The scenario file also includes three UN population projections: `UNMED2010`, `UNMED2012` and `UNMED2015`. Finally, the World Bank's GIDD projections are available with the label `GIDD`. The latter is intended to duplicate `UNMED2015`. It is only available through 2050.
5. **OVERLAYPOP**. This option allows to replace GTAP's base year population level with that from the scenario database. In any case, only the growth rates from the population projections are used from the initial base year level.<sup>16</sup>

<sup>14</sup> <http://www.rug.nl/ggdc/productivity/pwt/>

<sup>15</sup> The PIK projections have not been processed since they reflect a fixed aggregation of 32 regions, whereas the OECD and IIASA projections were done for most countries.

<sup>16</sup> There is currently an inconsistency in the population levels in the SSP database and GTAP. In the former, population is in levels and in the latter it is millions. This will affect the scaling of population and interact with

6. **TASS.** This option determines the specification for the aggregate land supply curve. Valid options are: **KELAS** for iso-elastic supply, **LOGIST** for logistic supply curve, **HYPERB** for hyperbola supply curve, and **INFTY** for infinitely elastic (i.e. horizontal) supply curve.
7. **WASS.** This option determines the specification for the aggregate water supply curve. It has the same valid options as the aggregate land supply curve.
8. **utility.** This option determines the specification of the household utility function. Five options are available: **CD** for the Cobb-Douglas utility function, **LES** for the linear expenditure system, **ELES** for the extended linear expenditure system, **AIDADS** for the AIDADS demand system, and **CDE** for the Constant-difference-in-elasticity demand system. The **CD**, **LES** and **AIDADS** are new options. The **LES** is calibrated to the Frisch parameter, which is currently parameterized in the calibration routine (`cal.gms`), but it would be preferable to have this entered as user input. The **AIDADS** is coded, but there is no calibration. It has been tested assuming the **LES** as one of the special cases of **AIDADS**. The Cobb-Douglas simply uses the base year budget shares to calibrate the marginal budget shares and the subsistence minima are set to zero.
9. **NRITER.** This option controls the number of iterations when running the model convergence in single country mode. As this has not been tested with the latest version of the model, it is best to set this option to 0.
10. **savfFlag.** This option controls the capital account closure. Two options are currently available. A value of **capFix** uses the fixed capital account closure rule. A value of **capFlex** uses the flexible capital account closure rule that allocates global savings in order to equalize expected returns across regions.

The remaining global options are entered as scalars. The first set provide the context for the model: comparative static and recursive dynamic with or without dynamic calibration.

1. **ifDyn.** This option takes two values. A value of 0 indicates a comparative static model. A value of 1 indicates a dynamic model. In the standard package, this will be set automatically when the simulation is invoked and the user sets the **simType** option. Valid values are **CompStat** and **RcvDyn**.
2. **ifCal.** This option takes two values and is only meaningful for dynamic scenarios. A value of 0 runs a dynamic scenario with pre-calibrated trends (generated by a baseline). A value of 1 runs a dynamic calibration scenario where specific dynamic trends are calibrated to exogenous indicators such as GDP. This value will be set in dynamic variables by setting the **ifCal** option when invoking the simulation.
3. **ifVint.** This options takes the value 0 or 1. It is set automatically when the simulation is invoked by the user. A value of 1 implements vintage capital, which is valid only for dynamic simulations. It should be set to 0 for comparative static simulations.

The remaining global options are typically invariant across simulations. The following list enumerates the options.

---

the **OVERLAYPOP** option. A default value for the population scale is  $10^{-6}$ . In the case of using the GTAP level, this will result in population levels being in the trillion, whereas if using the overlay, population levels will be in the million. This probably has no bearing on the model scaling, but should be verified. It would be best to unify the population scales across databases.

1. **inScale**. Scale factor for input data. The input SAM is typically in millions. A scale of  $10^{-6}$  has been found to be a useful scaling factor.
2. **outScale**. Scale factor for output data. This is typically just the inverse of **inScale**.
3. **popScale**. Scale factor for population. Often set to  $10^{-6}$ , though need to check on the population level inconsistencies.
4. **IScale**. Scale factor for labor volumes. [Need to verify scale in 'wages' database.]
5. **eScale**. Scale factor for energy. The energy volumes are in million tons of oil equivalent (MTOE). A scale factor of 0.001 is typically used.
6. **watScale**. Scale factor for water, typically  $10^{-12}$  is used. [Check units of volume database.]
7. **cScale**. Scale factor for emissions, typically 0.001 is used. [Check units for emissions.]
8. **ifCEQ**. Convert emissions to CEq. Input emissions are in CO<sub>2</sub>eq. If this flag is set to 1, the units will be converted to Ceq. The climate module [tbd] is based on emissions in Ceq.
9. **ArmFlag**. Set to 1 for agent-based Armington. If the flag is set to 0, the top level Armington sourcing is done at the aggregate level. [This is a new option.]
10. **ifNRG**. Set to 1 to use energy volumes. This will provide a volume/price split for energy commodities.
11. **ifNRGNest**. Set to 1 for energy nesting. A value of 0 will have only a single nested energy nest.
12. **ifMCP**. Set to 1 for MCP. A value of 0 will use NLP. The objective function is Walras' Law.
13. **ifLandCET**. Set to 1 to use CET for land allocation. A value of 0 will use the additive CET specification.
14. **ifSUB**. Set to 1 to substitute out equations. A value of 0 will have the full model specification without substitution. This will considerably increase the size of the model.
15. **IFPOWER**. Set to 1 for power module, which requires the power database, else set to 0.
16. **IFWATER**. Set to 1 for water module, which requires the water database, else set to 0
17. **ifAggTrade**. Set to 1 to aggregate trade in SAM. A value of 0 will have the full bilateral trade matrices output as part of the SAM.
18. **skLabgrwgt**. Set to between 0 and 1. This determines the growth assumptions for skilled and unskilled labor. The growth of skilled labor is driven by the growth of the appropriate education categories. The growth of unskilled labor will then be calculated by residual since the growth of overall labor will be determined by the growth of the working age population (15-64). One can use this parameter to modulate this calculation. If the weight is set to 1, then the growth rate of skilled labor will match exactly the growth of the corresponding education categories, ignoring any resulting impact on the growth of unskilled labor. At the other extreme, if the weight is set to 0, the growth of both skilled and unskilled will be identical and equal to the growth rate of total labor, i.e. the education profiles are fully ignored. Users can choose an intermediate value between 0 and 1.



## D.2.4 Comparative static simulations

In the current setup, there is very little difference between running comparative static and simulations dynamic simulations. Both the `runsim.gms` and `[basename]Opt.gms` files are coded to run both types of simulations. One of the key differences between the two is the time framework. The standard comparative static simulation has three 'time' periods labeled `base`, `check` and `shock`. The user is free to modify these in the 'Opt' file. One of the primary purposes of the comparative static version of the model is to test model initialization, calibration and homogeneity. It is also extremely useful to test new model specifications or parameterizations. With a new model version—for example a new aggregation, new specification or parameterization, it is always recommended practice to do a full diagnostic check.

A standard diagnostic check is to run the comparative static model with a single homogeneity shock in the `shock` time period. The `base` year is never run, it is meant to be able to replicate the initial database. The `check` simulation replicates `base` if the initialization and calibration is working correctly. For this reason, it is important to look at the largest residual in the listing file (or console). Open the listing file and do a search for `LOOPS`. This should take you to the diagnostics for the first simulation, i.e. the `check` simulation. Model diagnostics will appear by scrolling down—for example model size. The key diagnostic is `INITIAL POINT STATISTICS`. The maximum of `F` should be 0 or near zero. The largest error should reflect more or less the precision of the input SAM. One may also want to observe the initial Jacobian to see the range of the minimum and maximum elements. The range should be relatively narrow. A good maximum should be no larger than 1.0e003. A larger number is an indication of a scaling issue.

A second check of the model is to look at the values for the `LHS` in the list code, which stands for the left-hand side. This is controlled by the `LIMROW` option. It may default to either 0 or 3. In the case of 0, no equations will be listed. In the case of 3, up to 3 in any block will be listed. Setting `LIMROW` to a higher number will increase the number of equations printed in a block, for example: `Options limrow=100 ;`. The equations listing shows the value of the residual of an equation, i.e. it collects all endogenous variables on the left-hand side and evaluates the expression and it collects all additive exogenous variables and parameters on the right-hand side and evaluates the right-hand side expression. It then displays the value of the left-hand side that should match the displayed right-hand side. In most cases, the right-hand side will evaluate to 0, but it does not have to.<sup>17</sup> If there is a serious initialization/calibration problem, it may be necessary to increase `limrow` and visually go through the equation listings. For a quick inspection, do a search for `LHS` to find the first equation in the listing file. Then search for `****`. The four asterisks indicate an 'infeasible' solution. Note that in most cases, the infeasibility is likely to be a very small number—and typically reflects the accounting precision of the incoming SAM. To avoid nuisance infeasibility, one can increase the tolerance level by using the `tolinfrep` attribute of a model. In the default settings, the infeasibility tolerance has been set to 1.0e-005, and with a well-balanced SAM, this will generally avoid any listed infeasibilities.

The homogeneity test requires a shock file. The shock file should have the same name as the simulation. Thus if the simulation is assigned the name `COMP`, the user should create the a file named `COMPShk.gms`. Listing D.14 illustrates one way of implementing the homogeneity shock. It tests the value of the time period, and then increases the value of the exogenous numéraire by 50%.

Listing D.14: Homogeneity shock

---

```

2 if(sameas(tsim,"shock"),
3   pnum.fx(tsim) = 1.5*pnum.l(tsim) ;

```

<sup>17</sup> For example the labor equilibrium condition will have the exogenous supply of labor on the right-hand side.

4 ) ;

It is useful to do a quick check of the diagnostics by loading the output CSV file into Excel—and most convenient in Excel’s pivot tables. The first thing to look at is the resulting SAM’s. All accounts should be perfectly balanced for each individual region and for each time period. The second diagnostic is to compare the **base** values with the **check** values. They should be identical to within very narrow margins. A third check is to compare the **check** values with the **shock** values. All volume variables should be identical and all value variables and prices should have increased by the same percentage amount as the numéraire, for example 50%.

These are minimal diagnostics, but the user may want to do others—particularly if there is a change in specification. These would include shocks to taxes, for example tariffs, shock to factor stocks, productivity shocks, etc.

The distribution comes with an option file for the ‘10x10’ aggregation, as well as the generic `runsim.gms` file. The latter is set to handle many different situations but users may find it useful and/or necessary to make a copy of it and insert modifications. The distribution also comes with a command file that is needed to run the file in a Windows console. To run the homogeneity test, the command line could be the following:

```
runsim Homog Comp CompStat 0
```

The first argument will be the simulation name. In comparative static simulations the second argument is ignored, but must be present. In dynamic simulations it is the name of the baseline file. The third argument must be **CompStat** for comparative static simulations. The fourth argument is ignored for comparative static simulations, it is used for dynamic simulations. It is best to set it to zero.

Users will need to modify the `runsim` command file for their installations. An example is depicted below.

```
gams runSim --simName=%1 --BauName=%2 --simType=%3 --ifCal=%4 --baseName=10x10
--odir=v:\Output\EnvLink\10x10 -idir=..\model5n
-scrdir=v:\Output\EnvLink\10x10 -ps=9999 -pw=150
```

The `runSim` command file expects four arguments as described above. It is setup for a specific project. In this case it is setup for a project named ‘10x10’. The user needs to specify a folder for the output directory. Use ‘.’ to specify the current directory. It is also required to specify the model directory. In the example above, the model directory is at the same level as the simulation directory and named `model5n`. The remaining parameters are optional and the user is free to add others.<sup>18</sup>

<sup>18</sup> Simulation output can be voluminous depending on the aggregation and the time span of a simulation. With increasing use of backups and the cloud, it is sometimes useful to store output on non-critical storage in order to minimize bandwidth problems and exceeding allowed storage capacity.

## D.2.5 Dynamic simulations

Dynamic simulations are in principle only somewhat more complicated than comparative static simulations—though in practice raise considerably more problems. There are two key differences. The first is the time dimension. This is under the discretion of the user. For GTAP V9, the starting year should be set at 2011. The terminal year and the intermediate years are at full discretion. The model is setup to handle year step sizes of more than 1 [though this probably needs additional testing since it hasn't been used in a while]. One potential problem with multi-year time steps could be convergence, though this has proven to be less of a problem further out when the model has reached some sort of steady state. The second key difference is the implementation of the vintage capital formulation. This in principle should be transparent to the user.

In principle there is nothing to change in the 'Opt' file nor in the `runsim.gms` file. There are default statements that are implemented in the file `iterloop.gms` that generate the necessary closures for dynamic simulations—either the baseline, or pre-calibrated scenarios. Users have some control in shaping the baseline by modifying or adding statements in the `BaUShk.gms` file. In the distributed file, there are two adjustments to the standard baseline. The first phases out net capital flows between two years.<sup>19</sup> The second targets the investment to GDP ratio for some given future year—both the target and the year are provided by the user. It endogenizes the savings rate in order to achieve the desired investment ratio.

Other key dynamic assumptions are embedded in the `initScen.gms` file that is in the *Model* directory. In particular the default assumptions on labor productivity wedges (`glAddShft` and `glMltShft`), autonomous yield growth (`yexo`), autonomous energy efficiency improvement (`aeei` and `aeec`) and improvements in trade margins (`tteff`) are included in this file.[We may revisit this looking ahead as it is not satisfactory to have these assumptions included in the core model code.]

The mechanics of the baseline are fairly straightforward—shaping the baseline is not always straightforward. One of the key issues to arise is the growth of the capital stock. Base year conditions may lead to extreme jumping off conditions. For example over-investment may lead to sharply declining rates of return, and under-investment the reverse. The initial growth of the capital stock will depend on base year investment (and savings) rates, net capital flows, the initial stock of capital and the depreciation rate. On balance, one would like to have as a starting point for the baseline relatively steady returns to capital, unless there is a good reason to expect rates of return to rise or drop. It may involve adjusting the initial capital stock and rates of depreciation. It is not possible to change the initial savings rate including the capital account, but these can be adjusted over time to yield a desired path for capital accumulation and rates of return. This is one of the purposes for using a long-run target for the investment to GDP.

To run the baseline simulation, the command line could be the following:

```
runsim BaUSSP2 BaUSSP2 RcvDyn 1
```

The first argument and second arguments are the name of the baseline. In the case of running the baseline, the second argument is redundant, but necessary. The third argument is to invoke a dynamic-type scenario. The fourth argument invokes the code for dynamic calibration. Quite frequently, the model will fail to converge to the terminal year and typically this is due to factor prices going to some extreme—notably zero for capital in the case of investment exceeding demand for capital. One may have to restrict the simulation to an intermediate year and assess intermediate

<sup>19</sup> The current phase-out years are set to the final year, so these statements have no impact.

results to understand the nature of the problem. This can be done by restricting the time loop, for example `loop(tsim$(years(tsim) le 2020)`.

Once the user has a satisfactory baseline, dynamic shock simulations can be implemented. They are associated with a specific shock file with a base name linked to the simulation name, `[simName]Shk.gms`, for example `[SSP2ccShk.gms]` to run the baseline SSP2 with climate change impacts. To run a pre-calibrated dynamic scenario, the command line could be the following:

```
runsim SSP2CC BaUSSP2 RcvDyn 0
```

The first argument is the name of the simulation, and `runSim.gms` will try and open and implement a shock file with a filename that incorporates the simulation name. The second is the name of the baseline file. This is necessary for two reasons. The first is that pre-calibrated scenarios will load the pre-calibrated trends from the baseline file, for example labor productivity. The second reason is that under certain circumstances, a pre-calibrated scenario will use the solution from the baseline as a starting point. In fact, if the following command is issued without the appropriately named shock file, the simulation should run incredibly quickly since it uses the baseline as a starting point:

```
runsim noShock BaUSSP2 RcvDyn 0
```

There is one caveat—it requires modifying the `runSim.gms` file. The `iterloop` `batinclude` statement has an option that is normally a year. The argument tells `iterloop` how to initialize the variables for a new period. If the new period is less than the year that is the argument to `iterloop`, it will use the values from the baseline. Otherwise it uses the values from the previous period (with some adjustments.) For the baseline, the argument is set to 2011, as there is no baseline that can be used for initialization. For pre-calibrated scenarios, it can be set to any year between the initial and final year (inclusive). In the default `runSim.gms` file, the argument is set to 2020, because the underlying assumption is that any shock will occur in 2020 or later. For the *noShock* scenario, it should be set to the terminal year, for example 2050. If the dynamics is set up correctly, the model should solve within 1 iteration in each period. In effect, the *noShock* scenario is similar in concept to the *Check* scenario for comparative statics. The rule of thumb is that you can use the baseline till the first year of a shock. In most cases, initializing with the previous period's result is better once the shock has been implemented as the shock tends to lead to sharp deviations from the baseline.

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